

Homework

Lagrange's interpolation.

1)	x	$f(x)$	$x_n = 10$	$\nabla^2 f(x)$	$\nabla^2 f(x)$	$\nabla^3 f(x)$
	7	3	$f(x_n) = 9$	-2	+2	6
	8	1	$h = 1$	0	8	$\nabla^4 f(x)$
	9	1	$u = \frac{9.5-10}{h}$	8	$\nabla^3 f(x)$	
	10	9	$= -0.5$	$\nabla^2 f(x)$		

$$\therefore f(x) = f(x_n) + u \cdot \nabla f(x) + \frac{u(u-1) \cdot \nabla^2 f(x)}{2!} + \frac{u(u-1)(u-2) \nabla^3 f(x)}{3!}$$

$$f(9.5) = 9 + (-0.5)(8) + \frac{(-0.5)(-1.5) \cdot 8}{2} + \frac{(-0.5)(-1.5)(-2.5)}{6}$$

$$= 9 - 4 + 3 - 1.875$$

$$= 6.125$$

2)	x	y
	3	168
	7	120
	9	72
	10	63

$$\therefore f(6) = \frac{(6-7)(6-9)(6-10)}{(3-7)(3-9)(3-10)} \times 168 + \frac{(6-3)(6-9)(6-10)}{(7-3)(7-9)(7-10)} \times 120$$

$$+ \frac{(6-3)(6-7)(6-10)(72)}{(9-3)(9-7)(9-10)} + \frac{(6-3)(6-7)(6-9)(163)}{(10-3)(10-7)(10-9)}$$

$$\therefore f(6) = 12 + 180 - 72 + 127$$

$$= 147$$

3)	x	1	1	2	7	8
	$f(x)$		4	5	5	4

$$f(6) = \frac{(6-2)(6-7)(6-8)}{(1-2)(1-7)(1-8)} (4) + \frac{(6-1)(6-7)(6-8)}{(2-1)(2-7)(2-8)} (5)$$

$$+ \frac{(6-1)(6-2)(6-8)}{(7-1)(7-2)(7-8)} (5) + \frac{(6-1)(6-2)(6-7)}{(8-1)(8-2)(8-7)} (4)$$

$$= \frac{-16}{21} + \frac{10}{6} + \frac{10}{3} - \frac{40}{21}$$

$$= 2.333$$

4)	x	1	2	3	4	7
	$f(x)$	2	4	8	16	128

$$f(5) = \frac{(5-2)(5-3)(5-4)(5-7)}{(1-2)(1-3)(1-4)(1-7)} (2) + \frac{(5-1)(5-3)(5-4)(5-7)}{(2-1)(2-3)(2-4)(2-7)} (4)$$

$$+ \frac{(5-1)(5-2)(5-4)(5-7)}{(3-1)(3-2)(3-4)(3-7)} (8) + \frac{(5-1)(5-2)(5-3)(5-7)}{(4-1)(4-2)(4-3)(4-7)} (16)$$

$$+ \frac{(5-1)(5-2)(5-3)(5-4)}{(7-1)(7-2)(7-3)(7-4)} (128)$$

$$= \frac{1}{3} + \frac{32}{5} - 6 + \frac{64}{3} + \frac{128}{15}$$

$$= 30.6$$

5)

x	10	12	14	16
$f(x)$	0.25	0.2	0.15	0.1

$$f(15) = \frac{(15-12)(15-14)(15-16)}{(10-12)(10-14)(10-16)} (0.25) + \frac{(15-10)(15-14)(15-16)}{(12-10)(12-14)(12-16)} (0.2)$$

$$+ \frac{(15-10)(15-12)(15-16)}{(14-10)(14-12)(14-16)} (0.15) + \frac{(15-10)(15-12)(15-14)}{(16-10)(16-12)(16-14)} (0.1)$$

$$= \frac{0.25}{16} - \frac{0.5}{8} + \frac{2.25}{16} + \frac{1.5}{48}$$

$$= 0.125$$

15 ~~Hammer~~ Regular False Method.

since,

x	0.5	1
y	-0.375	1

$$x^3 + x - 1 = 0$$

lying between 0.5 & 1.

$$20 \quad \bar{x} = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1} = \frac{0.5(1) - (-0.375)(1)}{1 - (-0.375)}$$

$$= 0.6364$$

$$25 \quad \bar{y} = x^3 + x - 1$$

$$= (0.6364)^3 + (0.6364) - 1$$

$$= -0.1059$$

30

x	0.6364	1
y	-0.1059	1

$$\bar{x} = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1} = \frac{0.6364(1) - (-0.1059)(1)}{1 - (-0.1059)} = 0.7009$$

$$\begin{aligned}\bar{y} &= x^3 + x - 1 \\ &= (0.7009)^3 + (0.7009) - 1 \\ &= 0.0452\end{aligned}$$

7094

x	0.6364	1
y	0.0452	1

$$\bar{x} = \frac{0.6364(0.0452) - (-0.1059)(0.7009)}{0.0452 - (-0.1059)} = 0.6816$$

10

$$\begin{aligned}\bar{y} &= (0.6816)^3 + 0.6816 - 1 \\ &= -0.0017\end{aligned}$$

∴ ~~0.68~~ 0.6816 is the root of $x^3 + x - 1 = 0$.

15

2) $x^3 - 2x + 0.5 = 0$ lying between 0 & 1.

Since,

x	0	1
y	0.5	-0.5

20

$$\bar{x} = \frac{0(-0.5) - 1(0.5)}{-0.5 - 0.5} = 0.5$$

25

$$\begin{aligned}\bar{y} &= (0.5)^3 - 2(0.5) + 0.5 \\ &= -0.375\end{aligned}$$

7094

x	0	0.5
y	0.5	-0.375

30

$$\bar{x} = \frac{0(-0.375) - (0.5)(0.5)}{-0.375 - 0.5} = 0.2857$$

$$\begin{aligned}\bar{y} &= (0.2857)^3 - 2(0.2857) + 0.5 \\ &= -0.0480\end{aligned}$$

Jan

x	0	0.2857
y	0.5	-0.0480

$$\bar{x} = \frac{0(-0.0480) - (0.5)(0.2857)}{-0.0480 - 0.5} = 0.2607$$

$$\bar{y} = (0.2607)^3 - 2(0.2607) + 0.5 = -0.00368$$

10 $\therefore 0.2607$ is a root of $x^3 - 2x + 0.5 = 0$.

3)

x	2	2.5
y	-8	0.625

$$\bar{x} = \frac{2(0.625) - (8)(2.5)}{0.625 - (-8)} = 2.4638$$

$$\bar{y} = (2.4638)^3 + 2(2.4638) - 20 = -0.1164$$

Jan

x	2.4638	2.5
y	-0.1164	0.625

$$\bar{x} = \frac{2.4638(0.625) - 2.5(-0.1164)}{0.625 - (-0.1164)} = 2.4695$$

$$\bar{y} = (2.4695)^3 + 2(2.4695) - 20 = -0.00092649$$

30 $\therefore 2.4695$ is a root of $x^3 + 2x - 20 = 0$

4) $3x - \cos x - 1 = 0$ lying between 0 & 1

x	0	1
y	-2	1.0002

$$\bar{x} = \frac{0(1.0002) - (-2)(1)}{3.0002} = 0.6666$$

$$y = 3(0.666) - \cos(0.666) - 1 = 0$$

$\therefore x = 0.66$ is the root of the eqⁿ.

5) $x^3 - 3x - 5$ lying between 2 and 2.5.

Since

x	2	2.5
y	-3	3.125

$$\bar{x} = \frac{2(3.125) - (-3)(2.5)}{3.125 - (-3)} = 2.2449$$

$$\bar{y} = (2.2449)^3 - 3(2.2449) - 5 = -0.4214$$

x	2.2449	2.5
y	-0.4214	3.125

$$\bar{x} = \frac{2.2449(3.125) - (-0.4214)(2.5)}{3.125 - (-0.4214)} = 2.2752$$

$$\bar{y} = (2.2752)^3 - 3(2.2752) - 5 = 0.0479$$

For	x	2.2752	- 0.0479
	y	2.5	3.125

$$\bar{x} = \frac{2.2752(3.125) - (-0.0479)(2.5)}{3.125 + 0.0479}$$

$$= 2.2786$$

$$\begin{aligned}\bar{y} &= (2.2786)^3 - 3(2.2786) - 5 \\ &= -0.0052\end{aligned}$$

$\therefore x \approx 2.2786$ is root of eqⁿ.

6) $x \log_{16} x - 1.2 = 0$

since

x	2	3
y	-0.5957	0.2314

$$\bar{x} = \frac{2(0.2314) - (-0.5957)(3)}{0.2314 - (-0.5957)} = 2.7210$$

$$\begin{aligned}\bar{y} &= 2.721 \log_{10} 2.721 - 1.2 \\ &= 0.0171\end{aligned}$$

For	x	2.7210	3
	y	-0.0171	0.2314

$$\bar{x} = \frac{2.721(0.2314) - (-0.0171)(3)}{0.2314 - (-0.0171)} = 2.7402$$

$$\begin{aligned}\bar{y} &= 2.7402 \log_{10} 2.7402 - 1.2 \\ &= 0.000389\end{aligned}$$

$\therefore x \approx 2.7402$ is the root of the eqⁿ

7)

x	0	1
y	1	-2

$$\bar{x} = \frac{0(-2) - (1)(1)}{-2 - 1} = \frac{+1}{+3} = 0.\overline{33}$$

$$\therefore \bar{y} = (0.\overline{33})^3 - 4(0.\overline{33}) + 1$$

$$= -0.2962$$

For

x	0	$0.\overline{33}$
y	1	-0.2962

$$\bar{x} = \frac{2(-0.2962) - 1(0.\overline{33})}{-0.2962 - 1} = 0.2571$$

$$\bar{y} = (0.2571)^3 - 4(0.2571) + 1$$

$$= -0.0114$$

For

x	0	0.2571
y	1	-0.014

$$\bar{x} = \frac{0(-0.014) - (1)(0.2571)}{-0.014 - 1} = 0.2542$$

$$\bar{y} = (0.2542)^3 - 4(0.2542) + 1$$

$$= -0.00037$$

$\therefore x \approx 0.2542$ is the root of equation.

8)

x	0	1	2
y	-8	-5	8

$$\bar{x} = \frac{1(8) - 5(2)}{8 + 5} = \frac{8 - 10}{13} = -1.3846$$

$$\bar{y} = (1.3846)^3 + 2(1.3846)^2 - 8 = 1.5113$$

For,

x	1.3846	2
y	-1.5113	8

$$\bar{x} = \frac{(1.3846)(8) - 2(-1.5113)}{8 + 1.5113} = 1.4824$$

$$\bar{y} = (1.4824)^3 + 2(1.4824)^2 - 8 = -0.3474$$

For,

x	1.4824	2
y	-0.3474	8

$$\bar{x} = \frac{1.4824(8) - 2(-0.3474)}{8 + 0.3474} = 1.5039$$

$$\bar{y} = (1.5039)^3 + 2(1.5039)^2 - 8 = -0.0752$$

For,

x	1.5039	2
y	-0.0162	8

$$\bar{X} = \frac{1.5085(8) - (-0.0162)(2)}{8 + 0.0162} = 1.5095$$

$$\bar{Y} = (1.5095)^3 + 2(1.5095)^2 - 3 = -0.0033$$

$\therefore x \approx 1.5095$ is the root of the eqⁿ.

9) ₁₀

x	0	1
y	1	-7

$$\bar{X} = \frac{0(-7) - (1)(1)}{-7 - 1} = \frac{-1}{-8} = 0.125$$

$$\bar{Y} = (0.125)^3 - 9(0.125) + 1 = -0.1230$$

10) ₂₀

x	0	0.125
y	1	-0.1230

$$\bar{X} = \frac{0(-0.1230) - (1)(0.125)}{-0.123 - 1} = 0.1113$$

$$\bar{Y} = (0.1113)^3 - 9(0.1113) + 1 = -0.0003$$

$\therefore x \approx 0.1113$ is the root of the eqⁿ.

10)

x	0	1	2
y	-4	-4	2

$$\bar{x} = \frac{1(2) - 4(2)}{2 + 4} = 1.6667$$

$$\bar{y} = (1.6667)^3 - (1.6667) - 4 = -1.0368$$

For,

10

x	1.6667	2
y	-1.0368	2

$$\bar{x} = \frac{1.6667(2) - (-1.0368)(2)}{2 + 1.0368} = 1.7805$$

$$\bar{y} = (1.7805)^3 - 1.7805 - 4 = -0.1360$$

For,

20

x	1.7805	2
y	-0.1360	2

$$\bar{x} = \frac{1.7805(2) - (-0.1360)(2)}{2 + 0.136} = 1.7945$$

$$\bar{y} = (1.7945)^3 - (1.7945) - 4 = -0.0158$$

30 $\therefore x \approx 1.79$ is the root of the eqⁿ.