Maths assignment :

1) $\lim\limits_{h \to 0} \dfrac{2(-3+h)^2 - 18}{h}$

$\lim\limits_{h \to 0} \dfrac{2(h^2 - 6h + 9) - 18}{h}$

$\lim\limits_{h \to 0} \dfrac{2h^2 - 12h + 18 - 18}{h} \Rightarrow \dfrac{2\cancel{h}(h-6)}{\cancel{h}}$

$\lim\limits_{h \to 0} -12$

2) $g(x) = \begin{cases} 2x, & x < 6 \\ x - 1, & x \geq 6 \end{cases}$

(a) $x = 4$.

RHL: $\lim\limits_{x \to 4^+} x - 1 = 4 - 1 = 3$.

LHL: $\lim\limits_{x \to 4^-} 2(4) = 8$

RHL $\neq$ LHL so function is discontinuos at 4.

(b) $x = 6$

RHL: $\lim\limits_{x \to 6^+} x - 1 = 5$

$\lim\limits_{x \to 6^-} 2(6) = 12$

so it is ~~nit~~ discontinuos at 6.

3) $\lim_{x \to 9} \dfrac{x-9}{3-\sqrt{x}}$

$\lim_{x \to 9} \dfrac{x-9}{3-\sqrt{x}} \times \dfrac{(3+\sqrt{x})}{(3+\sqrt{x})}$

$\lim_{x \to 9} \dfrac{(x-9)(3+\sqrt{x})}{9-x}$.

$\lim_{x \to 9} - \dfrac{\cancel{(9-x)}(3+\sqrt{x})}{\cancel{(9-x)}}$

$\lim_{x \to 9} -6$

4) $f(x) = \dfrac{4x+5}{9-3x}$, (continuos if: $\lim_{x \to a} f(x) = f(a)$)

(a) $x = -1$.

$\lim_{x \to 1} \dfrac{(-4)+5}{9-(-3)} = \dfrac{1}{12}$, $f(-1) = \dfrac{1}{2}$.

so its continous

(b) $x = 0$.

$\lim_{x \to 0} \dfrac{4x+5}{9-3x} = \dfrac{5}{9}$ & $f(0) = \dfrac{5}{9}$

so its continous

c) $x = 3$

$\lim_{x \to 3} \dfrac{4x+5}{9-3x} = \dfrac{12+5}{9-9} = \dfrac{17}{0} \Rightarrow$ not defined

so it isn't continous at $x = 3$.

5) $$\lim_{x\to\infty} \frac{6e^{4x} - e^{-2x}}{8e^{4x} - e^{2x} + 3e^{-x}}$$

$$\Rightarrow \lim_{x\to\infty} \frac{\frac{6e^{4x}}{e^{4x}} - \frac{e^{-2x}}{e^{4x}}}{\frac{8e^{4x}}{e^{4x}} - \frac{e^{2x}}{e^{4x}} + \frac{3e^{-x}}{e^{4x}}}$$

$$\Rightarrow \lim_{x\to\infty} \frac{6 - e^{-6x}}{8 - e^{-2x} + 3e^{-5x}}$$

$$\Rightarrow \lim_{x\to\infty} \frac{6 - \frac{1}{e^{6x}}}{8 - \frac{1}{e^{2x}} + \frac{3}{e^{5x}}}$$

$$\Rightarrow \lim_{x\to\infty} \frac{6 - \frac{1}{\infty}}{8 - \frac{1}{\infty} + \frac{1}{\infty}} = \frac{6}{8} = \frac{3}{4}$$

6) $$g(y) = \begin{cases} y^2 + 5, & \text{if } y < -2 \\ 1 - 3y, & \text{if } y \geq -2 \end{cases}$$

1) $\lim_{y\to 6^-} g(y) \Rightarrow 36 + 5 = 41$

$\lim_{y\to 6^+} g(y) = 1 - 18 = -17$.

2) $\lim_{y\to 2^-} g(y) = 9y + 5 = 22.9$

$\lim_{y\to 2^+} g(y) = 1 - \cancel{8}\,6 = -5$

$\lim_{x\to -2} g(y)$ doesn't exi

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7) $f(t) = (4t^2 - t)(t^3 - 8t^2 + 12)$

Product rule : $uv' + u'v$

$f'(t) = (8t-1)(t^3 - 8t^2 + 12) + (4t^2 - t)(3t^2 - 16t)$

$= 8t^4 - 64t^3 + 96t - t^3 + 8t^2 - 12 + 12t^4 - 48t^3$
$- 3t^3 + 16t^2$

$= 20t^4 - 116t^3 + 24t^2 + 96t - 12$

8) $k(\omega) = \dfrac{3\omega + \omega^4}{2\omega^2 + 1}$.

Quotient rule : $\dfrac{vu' - uv'}{v^2}$

$\Rightarrow \dfrac{(2\omega^2+1)(3+4\omega^3) - (4\omega)(3\omega+\omega^4)}{(2\omega^2+1)^2}$.

$\Rightarrow \dfrac{6\omega^2 + 8\omega^5 + 3 + 4\omega^3 - (12\omega^2 + 4\omega^5)}{(2\omega^2+1)^2}$.

$\Rightarrow \dfrac{4\omega^5 + 4\omega^3 - 6\omega^2 + 3}{(2\omega^2+1)^2}$.

9) $f(x) = 2e^x - 8^x$

$= 2e^x - 8^x \log 8$.

10) $y = z^5 - e^z \ln(z)$

$\dfrac{dy}{dz} = 5z^4 - \left(\dfrac{e^z}{z} + e^z \ln(z)\right)$

$= 5z^4 - \dfrac{e^z}{z} + e^z \ln(z)$

11)

a) $f(x) = (6x^2+7x)^4$

$u = 6x^2+7x$.

$v = (u)^4$.

$$\frac{dy}{dx} = \frac{dv}{du} \times \frac{du}{dx}$$

$$= \frac{d(u)^4}{du} \times \frac{d(6x^2+7x)}{dx}.$$

$$= \cancel{4u}\ 4(6x^2+7x)^3 \times (12x+7)$$

$$= 4(6x^2+7x)(12x+7)$$

b) $f(t) = 5 + e^{4t+t^7}$.

$\cancel{u/x/4t/t^7}$

$$f'(t) = e^{4t+t^7}(4+7t^6)$$

$$= (7t^6+4)(e^{4t+t^7})$$

12)

a) $z = \ln(7-x^3)$

$$\frac{dz}{dx} = \frac{1}{7-x^3} \times -3x^2$$

$$\frac{d^2z}{dx^2} = \frac{(7-x^3)(-6x) - (-3x^2)(-3x^2)}{(7-x^3)^2}$$

$$\frac{d^2z}{dx^2} = \frac{(7-x^3)(-6x) - 9x^4}{(7-x^3)^2}$$

$$= \frac{-42x+6x^4-9x^4}{(7-x^3)^2}$$

$$= \frac{-42x-3x^4}{(7-x^3)^2}.$$

b) $\phi(v) = \dfrac{2}{(6+2v-v^2)^4}$

$\phi'(v) = 2(6+2v-v^2)^{-4}$

$= 2 \times -4(6+2v-v^2)^{-5}$

$= \dfrac{-8}{(6+2v-v^2)^5}$

$\phi''(v) = \dfrac{-40}{(6+2v-v^2)^6}.$

c) $g(t) = \ln(1+t^2)$

$g'(t) = \dfrac{2t}{1+t^2}$

$g''(t) = \dfrac{2(1+t^2) - 2t(2t)}{(1+t^2)^2}$

$= \dfrac{2+2t^2-4t^2}{(1+t^2)^2}$

$= \dfrac{2(1-2t^2)}{(1+t^2)^2}$

13) $x^3 - 3x^2 - 9x + 12 = f(x)$

$f'(x) = 3x^2 - 6x - 9$

Put $f'(x) = 0$

$0 = 3(x^2 - 2x - 3)$

$= 3(x^2 - 3x + x - 3)$

$= 3(x(x-3) + 1(x-3))$

$= 3(x+1)(x-3)$

$x = -1$ or $x = 3$

$f''(x) = 6x - 6$

case 1:

when $x = 3$

$f''(3) = 18 - 6 = 12$

$f''(3) > 0$

$f(x)$ is minimum at $x = 3$

$f(3) = (3)^3 - (3)(3)^2 - 9(3) + 12 = -15$ (Min value)

case 2:

$x = -1$.

$f''(-1) = -6 - 6 = -12$

$f''(-1) < 0$

$f(x)$ is max at $x = -1$.

$f(-1) = (-1) - 3(1) - 9(-1) + 12$

$= 17$ (Max value)

14) $4x^3 - 18x^2 + 24x - 7 = f(x)$

$f'(x) = 12x^2 - 36x + 24$

putting $f'(x) = 0$

$0 = 12(x^2 - 3x + 2)$

$= 12(x^2 - 2x - x + 2)$

$= 12(x(x-2) - 1(x-2))$

$= 12(x-1)(x-2)$

$x = 1 \quad , \quad x = 2$

$f''(x) = 24x - 36 = 12(2x-3)$

case 1 :

$f''(1) = 24(1) - 36 = -12$

$f''(1) < 0$

$f(x)$ is max at $x = 1$.

$f(1) = 4 - 18 + 24 - 7 = 3$ (Max value)

case 2 :

$f''(2) = 48 - 36 = 12$

$f''(2) > 0$

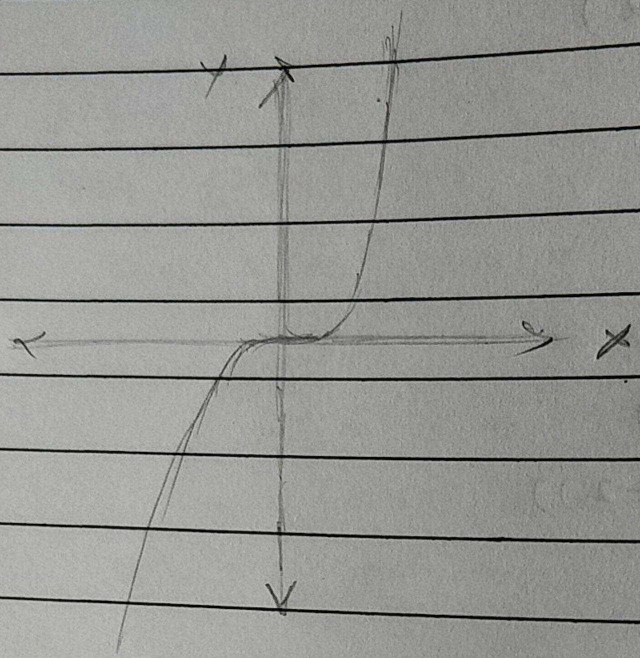
$f(x)$ is min at $x = 2$

$f(2) = 4(8) - 18(4) + 24(2) - 7$

$= 32 - 72 + 48 - 7$.

$= 1$ is min value.

15 Graph : $f(x) = x^2(x+3)$



16) $f(x) = 3x^2 - 6x$