

Dhruvi Shah

704

Pricing & Reserving for Life Insurance Products - 2

Assignment 2: Unit 3 & 4

1) There are numerous assumptions made when profit testing is done. For example about interest rate, risk discount rate; probability of mortality, surrenders etc.

All these assumptions are just predictions and may not reflect real life. Hence if we use sensitivity analysis i.e. use different values and scenarios to arrive at answer, we have a broader range of scenarios that we can expect which is more efficient.

2) Doubt -

3) i) Profit vector is the vector of balancing items in the projected revenue amounts for each policy year. It gives expected profit at the end of each policy year per policy in force at the beginning of that policy year.

ii) "Profit criterion" is the objective specified for expected level of profit. eg. Profit Margin should be 5% of the EPV of Premium Income or NPV is 50% of initial commission.

iii) discount rate = $r_f + r_m$
 $= 10\% + 3\% = 13\%$

$$\begin{aligned} \text{EPV profit} &= -250v + 150v^2 + 200v^3 + 225v^4 \\ @ 13\% &= 172.84 \end{aligned}$$

$$\begin{aligned} \text{EPV of Prem} &= P(1 + 1.04 \cdot 1/42v + 1.04^2 \cdot 2/42v^2 + 1.04^3 \cdot 3/42v^3) \\ @ 13\% &= P(1 + 0.91934 + 0.8457 + 0.7768) \\ &= 3.5412P \end{aligned}$$

$$\text{Profit Margin} = \frac{\text{EPV of Profit}}{\text{EPV of Prem}}$$

$$5\% = \frac{172.84}{3.5412P}$$

$$P = 976.17$$

Premium is 976.17

4) i) There are three bases experience basis, pricing basis and valuation (renewing) basis.

ii) The assumption of student actuary is correct that reserves are held such that there is an acceptably low probability of insolvency occurring in future but it is incorrect to assume that everyone will die on the day after the valuation date. This is possible because a negative perspective will lead company to hold high reserves which is high capital requirement. This will require higher profits resulting in higher premium/charge for customer.

$$(iii) \text{ Profit } V = [-20, \underset{1p[70]}{-35.20}, \underset{2p[70]}{50}, \underset{3p[70]}{-28.5}, \underset{4p[70]}{-20}, \underset{5p[70]}{90.21}]$$

$$= [-20, -35.79, 52.12, -30.65, -22.27, 104.4]$$

$$5V = 0$$

$$4V = \frac{22.27}{1.06} = 21.01$$

$$3V = \frac{(30.65 + 21.01 \cdot 1p[73])}{1.06}$$

$$= 48.06$$

$$2V = 0$$

$$= 52.12 - 48.06 \cdot 1p[72]$$

$$= 55.4$$

$$1V = \frac{35.79}{1.06} = 33.76$$

$$\text{Revised profit vector} = [-20, -33.76, 1p[70]] \\ = -53.2$$

Revised profit vector = $(-53.20, 0, 5.54, 0, 0, 104.4)$

5) i) Unit fund represents the value of unit of policyholder at any time. It does not necessarily represent the amount that the ph is entitled to at any time. On death, maturity or surrender, the units ~~not~~ held will be used to pay the benefit.

Sources of non-unit fund:

- premium less cost of allocation
- expenses
- interest
- FMC taken from unit fund
- cost of outgo
- Profit on surrender

ii) It is financially prudent to have a product that is self financing. The company sets up non unit reserve of the small cashflow in any month ~~and~~ other than the first is negative without reserves.

Reserves are set up early so that money can be released as & when required to eliminate the negative cashflow. Setting up of reserves will result in delay of profit but will be self financing.

6)

Year	i)	q_x	p_x	p_{x-1}	$(PS)_t$
1	Profit V				
	-250	0.00059	0.99941		(250)
2	-400	0.000602	0.999398	0.99941	(399.76)
3	-600	0.000617	0.999383	0.998808	(599.29)
4	1500	0.000636	0.999364	0.998192	1497.29

Present Value

(235.15)

(355.79)

(503.12)

1185.99

$$NPV = 91.18$$

$$ii) \quad 2V \quad 2V = -600 \quad 2-582.52$$

$$\frac{1.03}$$

$$1V = \frac{-450 + 582.52(0.9941)}{1.03}$$

$$= -953.57$$

$$\text{Year 1 CF} = -200 - 953.57 \times 0.99941$$

$$= -1203$$

$$NPV = \frac{-1203}{1.06} + 1185.99$$

$$= 57.08$$

iii) NPV after zeroing the cashflows is smaller. As negative cashflows have been accelerated hence being discounted less

F) 1) Decrement table

Age	q_x^d	q_x^s	(aq_x^s)	(aq_x^d)	$(ap)_x$	${}_t(ap)_{x-1}$
61	0.006433	0.06	0.05981	0.00624	0.93395	1
62	0.009696	0.06	0.05971	0.00941	0.93089	0.93395
63	0.011344	0.06	0.05966	0.011	0.92934	0.8694
64	0.012116	0.06	0.05962	0.01233	0.92805	0.80797

Unit fund cashflow						
Year	Prem	Fund at Start	All Prem	less B/o	pfce fund before Int	Int
1	30000	-				
2	30000	-	12000	600	11400	570
3	30000	11790	27000	1350	37440	187
4	30000	38723	30000	1500	67223	3361
5	30000	69525	33000	1600	100875	5048

Year	Fund after Int	less FMC	Fund at end
1	11970		
2	39312	180	11790
3	70584	590	38723
4	105919	1059	69525
5		1589	104330

Non unit fund cashflow								
Year	Prem - to	B/o Sp.	Com	Exp.	Fund before Int	Int	Maturity B	Fm
1	18000	600	6000	5000	7600	266		180
2	3000	1350	600	2000	1750	61.25		190
3		1500	600	2000	(1100)	(-38.5)		107
4	-3000	1650	600	2000	(3950)	(138.25)	4841.16	1589

Year	Add benefit	Cashflow
1	550.43	7495.12
2	576.32	1824.62
3	335.34	(415.08)
4		(7340.63)

Profit Margin

PV of profit = 3673.91

PV of premium = 98573.07

Profit Margin = 3.72%

- (ii) Profit Margin could be higher or lower depending on definition of surrender benefit. If a good penalty is imposed, profit margin could improve.
- (iii) To cover initial expenses -

$$8) P \ddot{a}_{61:4} = 500000 A_{61:4} + (100 + 0.025P) (\ddot{a}_{61:4} - 1) + 800$$

$$P \times 3.73 = 500000 \times 0.8564 + (100 + 0.025P) (3.73 - 1) + 800$$

Solving,

$$P = 117250$$

Year	Prem	e	opening R	li to bnd	death C	Mgt Surv C
1	117250.7681	800	0	5822.54	3216.5	5825.02
2	117250.7681	3031.27	116200	11520.97	4848	11612.52
3	117250.7681	3031.27	237910	17606.48	5672	17387.12
4	117250.7681	3031.27	365665	23994.23	6358	493642.

Year	Closing R	Profit M
1	109679.67	3552.12
2	223821.92	1658.04
3	343442.08	3234.78
4		3878.72

$$NPV \text{ of Profit Signature} = 9288.79$$

$$NPV \text{ of Premium} = 386618.04$$

$$\text{Profit Margin} = 2.4\%$$

9)

x	$(aq)_x^d$	$(aq)_x^s$	$(ap)_x$	$x-1 (ap)_x$
42	0.000922	0.099908	0.89917	1
43	0.001150	0.099885	0.898965	0.89917
44	0.001327	.	0.998673	0.808323

year	Unit fund				MC
	Unit fund at start	Allocated P	b/o spent	Int.	
1	0	3000	75	263.25	39.85
2	3148.4	8000	200	985.36	149.17
3	11784.59	9250	231.25	1872.3	283.45

year	unit fund at end
1	3148.4
2	11784.59
3	22392.19

year	Non Unit fund						
	Unallocated P	B/o	e	Comm	Int	AMC	Death
1	7000	75	2000	1000	285.25	39.85	15.54
2	2000	200	750	300	80.5	149.17	7.45
3	750	231.25	750	300	-4.81	283.45	

year	Maturity C	CF	prob	Discount	Profit EPV	Prem Sig
1	0	4384.56	1	0.884956	3	10000
2	0	1370.22	0.89917	0.783147		8991.7
3	3354.37	-3144.48	0.808323	0.69305	3083.46	8083.23

EPV of Profit = 3083.46

EPV of Prem = 24287.61

Profit Margin = 12.7%

10)

Unit Fund

Year	Paid	Cost	Plan fund	Fund after ch	Fund after mc	Prof. fee
1	17500	15750	15750	16852.5	84.26	500
2	27500	24750	41018.24	43889.52	219.45	500
3	27500	24750	67920.04	72674.47	363.37	500
4	27500	24750	96561.1	103320.38	576.6	500

Year	Fund after cf
1	16268.24
2	43170.07
3	71811.1
4	102303.78

Non Unit Fund

Year	Paid	C	Non unit i	FMC	fee	Death C	Prof. fee
1	9250	3500	402.5	84.26	500	780.98	6005.78
2	250	1100	-59.5	219.45	500	327.45	-577.5
3	250	1600	-59.5	363.37	500	0	-46.13
4	250	1100	-59.5	576.6	500	0	107.1