22/11/21

Subject : Financial Mathematics

Chapters 1, 2, 3, 4 (Unit 1 & 2)

Assignment 1

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1. $d^{(4)} = 8\%$

(i) $\delta = ?$

$$\left(1-\frac{d^{(4)}}{4}\right)^4 = (1-d)$$

$$\left(1-\frac{d^{(4)}}{4}\right)^4 = e^{-\delta}$$

$$\left(1-\frac{0.08}{4}\right)^4 = e^{-\delta}$$

$$e^{-\delta} = 0.92236816$$

$$\delta = -\ln 0.92236816$$

$$= 0.080810829$$

$$\boxed{\delta = 8.0810829\%}$$

(ii)

$$\left(1-\frac{d^{(4)}}{4}\right)^4 = (1-d)$$

$$\left(1-\frac{d^{(4)}}{4}\right)^{-4} = (1+i)$$

$$\left(1-\frac{0.08}{4}\right)^{-4} - 1 = i$$

$$\boxed{i = 0.084165785 = 8.4165785\%}$$

iii) $\left(1-\frac{d^{(4)}}{4}\right)^4 = (1-d)$ and

$$\left(1-\frac{d^{(12)}}{12}\right)^{12} = (1-d)$$

$$\therefore \left(1-\frac{d^{(4)}}{4}\right)^4 = \left(1-\frac{d^{(12)}}{12}\right)^{12}$$

$$\left(1-\frac{0.08}{\cancel{2}4}\right)^{\cancel{4}\,1/3} = \left(1-\frac{d^{(12)}}{12}\right)^{\cancel{12}}$$

$$0.9604 = 1-\frac{d^{(2)}}{12}$$

$$d^{(2)} = 0.0792$$

$$d^{(2)} = 7.92\%$$

$$0.993288388 = 1-\frac{d^{(12)}}{12}$$

$$\boxed{d^{(12)} = 8.0539339\%}$$

2\. ? $\delta(t) = 0.004t + 0.0002t^2$

i) $A.V = ₹1000$

$t = 10$ years

$P.V_0 = A.V_{10} \times v(0,10)$

$= 1000 \times e^{-\int_0^{10} \delta(t)\,dt}$

$= 1000 \times e^{-\int_0^{10} 0.004t + 0.0002t^2\,dt}$

$= 1000 \times e^{-\left[0.002t^2 + \frac{0.0002}{3}t^3\right]_0^{10}}$

$= 1000 \times e^{-\left[0.002(100) + \frac{0.0002}{3}(1000)\right]}$

$= 1000 \times e^{-[0.2 + 0.06667]}$

$= 1000 \times e^{-0.266667}$

$\boxed{= ₹765.9283384}$

ii) $A.V = (1+i)^n \times P.V$

$1000 = (1+i)^{10} \times 765.9283384$

$\frac{1000}{765.9283384} = (1+i)^{10}$

$(1+i)^{10} = 1.305605172$

$(1+i)^{\cancel{10}} = 1.027025404$

$i = 0.027025404$

$= \boxed{2.7025404\%}$

3. i) $d = iv$

given : d = effective annual rate of discount

i = effective annual rate of interest

$$R.H.S = iv$$

$$= \frac{d}{(1-d)} \times (1-d)$$

$$= d$$

$\therefore$ When we multiply effective annual rate of interest with v, we get d.

ii)

(a) $$\left(1 - \frac{d^{(2)}}{2}\right)^2 = \left(1 - \frac{d^{(4)}}{4}\right)^4$$

$$\left(1 - \frac{d^{(2)}}{2}\right)^{\cancel{2}} = \left(1 - 0.0225\right)^{\frac{2}{\cancel{4}}}$$

$$\boxed{d^{(2)} = 8.89875\%}$$

(b) $$\left(1 - \frac{d^{(2)}}{2}\right)^2 = \left(1 - \frac{d^{(1/2)}}{(1/2)}\right)^{1/2}$$

$$\left(1 - \frac{d^{(2)}}{2}\right)^2 = \left(1 - \frac{0.05}{1/2}\right)^{1/2}$$

$$d^{(2)} = 0.05199250 7$$

$$= \boxed{5.1992507\%}$$

iii) $n = \frac{91}{365}$ $i = 0.05$

$$(1+i)^n = (1-dn)^{-1}$$

$$(1+0.05)^{-91/365} = \left(1 - \frac{91}{365} \times d\right)$$

$$0.987909561 = \left(1 - \cancel{d}\, \frac{91}{365} \times d\right)$$

$$d = 0.04849461 9$$

$$= \boxed{4.84946 19\%}$$

4. i) effective annual rate of interest and discounting factor are equated as :

$$(1+i) = \frac{1}{(1-d)} \quad \left[A(t) = \frac{1}{V(t)}\right]$$

hence on simplifying we get

$$i = \frac{1}{1-d} - 1 = \frac{1-1+d}{1-d} = \frac{d}{1-d}$$

Discounting factor is $\frac{\cancel{1}}{\cancel{(1-d)}}$ $(1-d)$

So, L.H.S = d

$$R.H.S = iv = \frac{d}{(1-d)} \times (1-d)$$

$$= d = L.H.S$$

When we multiply amount i by a discounting factor then we get d.

ii) $i = 0.08 = 8\%$

a. $d^{(12)}$

$$(1+i) = \left(1 - \frac{d^{(12)}}{12}\right)^{-12}$$

$$(1.08)^{-1/12} = \left(1 - \frac{d^{(12)}}{12}\right)^{\cancel{-12}}$$

$$0.993607102 = 1 - \frac{d^{(12)}}{12}$$

$$\boxed{d^{(12)} = 7.6714776\%}$$

b. $i^{(365)}$

$$(1+i) = \left(1 + \frac{i^{(365)}}{365}\right)^{365}$$

$$(1+0.08)^{1/365} = \left(1 + \frac{i^{(365)}}{365}\right)$$

$$0.000210874 = \frac{i^{(365)}}{365}$$

$$\therefore \boxed{i^{(365)} = 7.6969155\%}$$

c. $(1+i) = e^{\delta}$

$$\delta = \ln(1+i)$$

$$= \ln(1+0.08)$$

$$= 0.07696104 1$$

$$= \boxed{7.696104 1\%}$$

d. $i^{(0.5)}$

$$(1+i) = \left(1 + \cancel{0}\ \frac{i^{(0.5)}}{0.5}\right)^{0.5}$$

$$(1+0.08) = \left(1 + \frac{i^{(0.5)}}{0.5}\right)^{0.5}$$

$$\frac{i^{0.5}}{0.5} = 0.1664$$

$$= 0.0832$$

$$= \boxed{8.32\%}$$

5. i) $d^{(4)} = 6\%$

a) $$(1+i) = \left(1 + \frac{i^{(\cancel{4}2)}}{2}\right)^2$$

$$\left(1 - \frac{d^{(4)}}{4}\right)^{-4} = \left(1 + \frac{i^{(2)}}{2}\right)^2$$

$$\left(1 - \frac{0.06}{4}\right)^{-2} = \left(1 + \frac{i^{(2)}}{2}\right)$$

$$i^{(2)}/2 = 0.030688758$$

$$i^{(2)} = 0.061377516$$

$$= \boxed{6.1377516\%}$$

b) $d^{(12)}$

$$\left(1-\frac{d^{(4)}}{4}\right)^{\cancel{4}} = \left(1-\frac{d^{(12)}}{12}\right)^{\frac{3}{\cancel{12}}}$$

$$\left(1-\frac{0.06}{4}\right)^{1/3} = \left(1-\frac{d^{(12)}}{12}\right)$$

$$\frac{d^{(12)}}{12} = 0.00502521$$

$$d^{(12)} = 0.060302525$$

$$= \boxed{6.0302525}$$

ii) Rs 200,000 — ? — Rs 2,20,000

a) 15th April 2005 — 17th July 2005 — 15th April 2006

$$A.V = 2,00,000\ A(0,1)$$

$$2,20,000 = 2,00,000 \times A(0,1)$$

$$A(0,1) = 1.1$$

$$(1+i)^n = 1.1 \qquad n=1$$

$$1+i = 1.1$$

$$i = 10\%$$

15
31
30
17
93

$$(1+i)^n = \frac{A.V}{P.V}$$

$$A.V = 2,00,000 \times (1+0.1)^{93/365}$$

$$\boxed{= Rs\ 2,00,470.5755}$$

$\therefore$ Amit pays Rs 2,00,470.5755 to terminate the contract.

6. i) a) Cash payment : 30% below retail price
Six month credit : 25% below retail price
Let retail price be 100

P.V = 30% below retail price
$= 100 - 30$
$= 70$

A.V = 25% below retail price
$= 100 - 25$
$= 75$

$$A.V(1-d) = P.V$$
$$75(1-d) = 70$$
$$1-d = \frac{70}{75}$$
$$= 0.93333$$
$$\boxed{d = 6.66667\%}$$

ii) $d^{(12)}$

$$(1-d) = \left(1-\frac{d^{(12)}}{12}\right)^{12}$$
$$(1-0.066667)^{1/12} = \left(1-\frac{d^{(12)}}{12}\right)$$
$$\frac{d^{(12)}}{12} = 0.00573291$$
$$\boxed{d^{(12)} = 6.8794917\%}$$

$i^{(2)} = ?$

$$\left(1+\frac{i^{(2)}}{2}\right)^{-2} = (1-d)$$

$$\left(1+\frac{i^{(2)}}{2}\right) = (1-0.06667)^{-1/2}$$

$$1+\frac{i^{(2)}}{2} = 1.035098339$$

$$i^{(2)} = 0.035098339 \times 2$$
$$= 0.070196678$$
$$\boxed{= 7.0196678\%}$$

$\delta = ?$

$$-\ln(1-d) = \delta$$

$$-\ln(1-0.06667) = \delta$$

$$\delta = -\ln(0.933333)$$
$$= -(-0.068992872)$$
$$= 0.068992872$$
$$= \boxed{6.8992872\%}$$

7. $n = 17$ months $= \frac{17}{12} = 1.416667$ yrs.

$P.V = ₹\,20,000$

$A.V = ₹\,26,000$

i) $\left(1+\frac{i^{(4)}}{4}\right)^4 = (1+i)$

$$\left(1+\frac{i^{(4)}}{4}\right)^4 = \frac{A.V}{P.V} = \frac{26,000}{20,000}$$

$$\left(1+\frac{i^{(4)}}{4}\right)^4 = 1.3$$

$$\frac{i^{(4)}}{4} = 0.067789972$$

$$i^{(4)} = 0.271159889$$

$$= \boxed{27.1159889}$$

ii) $\left(1+\frac{i^{(2)}}{2}\right)^2 = (1+i)$

$$\left(1+\frac{i^{(2)}}{2}\right)^2 = 1.3$$

$$\frac{i^{(2)}}{2} = 0.140175425$$

$$\boxed{i^{(2)} = 28.035085\%}$$

iii) $(1+i)^n = (1+ni_s)$ i_s = simple interest

$$(1+0.3)^{17/12} = \left(1+\frac{17}{12}i_s\right)$$

$$1.45017278 = \left(1+\frac{17}{12}i_s\right)$$

$$\frac{17}{12}i_s = 0.45017278$$

$$\boxed{i_s = 31.7769021\%}$$

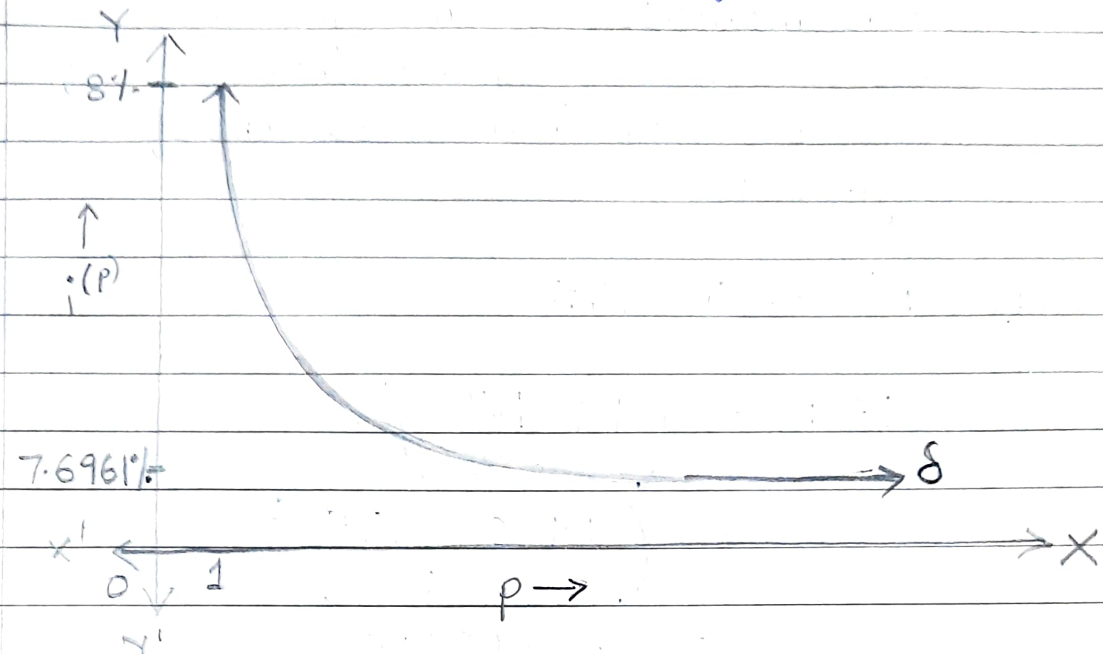
iv) As we know,

$$i > i^{(2)} > i^{(3)} \dots i^{(n)} > \delta > d^{(n)} > \dots d^{(2)} > d$$

Hence we got $i^{(4)}$ which was less as compared to $i^{(2)}$.
So, (ii) is higher than (i)

For a amount, for more than a year compound ~~year~~ interest leads to accumulating value higher than Accumulating value of simple interest So applying the same concept, if we keep accumulating value constant then simple interest has to be higher to get the accumulating value as compared to compounding.
Hence iii) is higher than (i) and (ii)

8. Relationship of effective interest rate of 8% with equivalent nominal rates of interest convertible pthly ($i^{(P)}$)



For an effective interest rate, say 8% p.a ~~sa~~ as given in question, the equivalent nominal rates of interest ~~decreases as~~ convertible pthly $i^{(P)}$ and p decreases as value of p increases. Also, the decrease in value of $i^{(P)}$ is substantial in the initial phase when p has smaller values but decrease in value gradually comes to a constant δ (delta) - which is force of constant at infinity.

9. i) Force of Interest :

Force of Interest measures the intensity with which the interest is operating each moment of time i.e over infinitesimally small interval of time or continuous stream of payments.

ii) $i^{(2)} = 0.0775$

$$(1+i) = \left(1+\frac{i^{(2)}}{2}\right)^2$$

$$= \left(1+\frac{0.0775}{2}\right)^2$$

$$= 1.079001563$$

$$i = 0.079001563$$

$$= \boxed{7.9001563\%}$$

$$\delta = \ln\left(1+\frac{i^{(2)}}{2}\right)^2$$

$$= \ln\left(1+\frac{0.0775}{2}\right)^2$$

$$= 0.076036134$$

$$= \boxed{7.6036134\%}$$

$d^{(12)} = ?$

$$\left(1-\frac{d^{(12)}}{12}\right)^{12} = \left(1+\frac{i^{(2)}}{2}\right)^{-2}$$

$$\left(1-\frac{d^{(12)}}{12}\right) = \left(1+\frac{0.0775}{2}\right)^{-1/6}$$

$$\left(1-\frac{d^{(12)}}{12}\right) = 0.993683688$$

$$d^{(12)} = 0.075795747$$

$$= \boxed{7.5795747\%}$$

$$i^{(1/2)} = ?$$

$$\left(1+\frac{i^{(2)}}{2}\right)^{2} = \left(1+\frac{i^{(1/2)}}{1/2}\right)^{1/2}$$

$$\left(1+\frac{0.0775}{2}\right)^{4} = \left(1+\frac{i^{(1/2)}}{1/2}\right)$$

$$0.164244372 = \frac{i^{(1/2)}}{1/2}$$

$$\boxed{i^{(1/2)} = 8.2122186\%}$$

10. $\delta(t) = \begin{cases} 0.08 \text{ for } & 0 \le t \le 5 \\ 0.06 \text{ for } & 5 \le t \le 10 \\ 0.04 \text{ for } & t \geq 10 \end{cases}$

case (i) : $0 \le t \le 5$

$P.V = 1 \times e^{-\int_0^t 0.08\, ds}$

$= e^{-[0.08s]_0^t}$

$= \boxed{e^{-0.08t}}$

case (ii) $5 \le t \le 10$

$P.V = 1 \times e^{-\int_0^5 0.08\, ds} \cdot e^{-\int_5^t 0.06\, ds}$

$= e^{-[0.08s]_0^5} \cdot e^{-[0.06s]_5^t}$

$= e^{-0.4} \cdot e^{-[\cancel{0.6}\,0.06t - 0.3]}$

$= e^{-0.4} \cdot e^{+0.3} \cdot e^{-\cancel{t}\,0.06t}$

$= e^{-0.\cancel{1} - t \times 0.06}$

$= \cancel{e^{-(t+0.1)}}$ $\boxed{e^{-(0.06t + 0.1)}}$

case (iii) $t \geq 10$

$P.V = 1 \times e^{-\int_0^5 0.08\, ds} \cdot e^{-\int_5^{10} 0.06\, ds} \cdot e^{-\int_{10}^t 0.04\, ds}$

$= e^{-0.4} \cdot e^{-0.3} \cdot e^{-[0.04s]_{10}^t}$

$= e^{-0.7} \cdot e^{-[0.04t - 0.4]}$

$= \boxed{e^{-[0.04t + \cancel{e}\,1.1]}}$

11. a) $n = 9$ month $= 9/12$ yr

$A.V = Rs\,50{,}000$

$d = 18\%$ p.a

$P.V = ?$

$$P.V = A.V \times V(t)$$
$$= A.V \times (1-d)^{n}$$
$$= 50{,}000 \times (1-0.18)^{0.75}$$
$$\boxed{= Rs\,43085.42623}$$

b) i) $\delta = 6\%$, $d^{(12)} = ?$

$$\left(1 - \frac{d^{(12)}}{12}\right)^{12} = e^{-\delta}$$

$$\left(1 - \frac{d^{(12)}}{12}\right)^{12} = e^{-0.06}$$

$$1 - \frac{d^{(12)}}{12} = 0.995012479$$

$$\frac{d^{(12)}}{12} = 0.004987521$$

$$d^{(12)} = 0.05985025$$
$$= \boxed{5.985025\%}$$

ii) $d^{(4)} = 6\%$

$$\left(1 - \frac{d^{(4)}}{4}\right)^{-4} = \left(1 + \frac{i^{(12)}}{12}\right)^{12}$$

$$\left(1 - \frac{0.06}{4}\right)^{-1/3} = 1 + \frac{i^{(12)}}{12}$$

$$\boxed{i^{(12)} = 6.0607089\%}$$

$i^{(4)} = ?$

$$\left(1+\frac{i^{(4)}}{4}\right)^{4} = \left(1-\frac{d^{(4)}}{4}\right)^{-4}$$

$$\left(\frac{i^{(4)}}{4}\right) = \left(1-\frac{0.06}{4}\right)^{-1} - 1$$

$$= 0.015228426$$

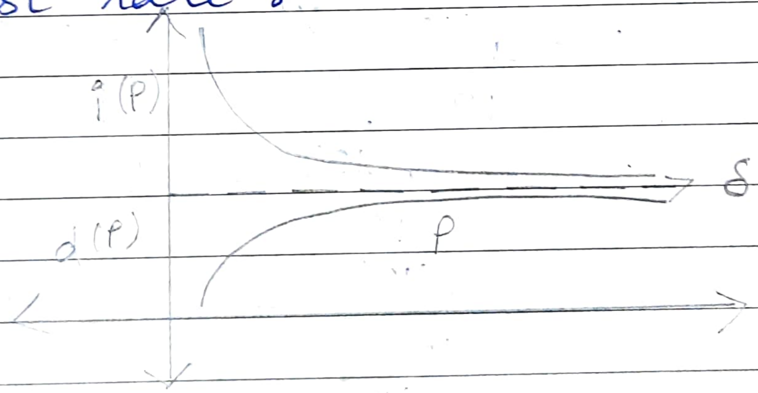
$$i^{(4)} = 0.060913706$$

$$\boxed{= 6.0913706\%}$$

c) Ascending order:

$$d < \delta < i^{(4)}$$

Explaination : For constant effective interest rate :



For $i^{(P)}$, the value decreases as value of P increases and leads to δ at infinity. So, $i^{(P)} > \delta$.
For $d^{(P)}$, the value increases as value of P increases and leads to δ at infinity. So, $d^{(P)} < \delta$

Hence, $d^{(P)} < \delta < i^{(P)}$

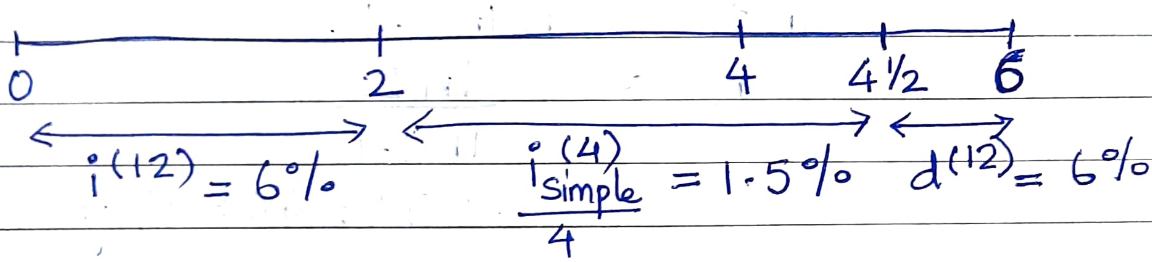
$\therefore d^{(1)} < \delta < i^{(4)}$

d) d and i are related as $i = \frac{d}{(1-d)}$

here $(1-d)$ denotes the discounting factor. So, $i = \frac{d}{v} \Rightarrow d = iv$

12\. 12)

a) P.V = Rs 7049



$$A.V^{(0,6)} = A(0,2) \times A(2,4.5) \times A(4.5,6)$$

$$A(0,6) = \left(1 + \frac{i^{(12)}}{12}\right)^{12\times2} \times \left(1 + n\frac{i^{(4)}}{4}\right) \times \left(1 - \frac{d^{(12)}}{12}\right)^{-12\times 1.5}$$

$$= \left(1 + \frac{0.06}{12}\right)^{24} \times \left(1 + (2.5\times4)\times 0.015\right) \times \left(1 - \frac{0.06}{12}\right)^{-18}$$

$= 1.127159776 \times 1.15 \times 1.094421325$

$= 1.418625849$

$A.V_6 = P.V \times A(0,6)$

$= 7049 \times 1.418625849$ $\boxed{= Rs\ 9999.893611}$

13. $2P.V = A.V \Rightarrow$ given $n = ?$

i) $i = 10\%$

$A.V = P.V \times (1+i)^n$

Put $A.V = 2P.V$ in eqn.

$2PV = P.V = (1+i)^n$

$2 = (1+0.1)^n$

$2 = 1.1^n$

taking ln on both sides

$\ln 2 = n \ln 1.1$

$n = \dfrac{\ln 2}{\ln 1.1}$

$= \boxed{7.272540897 \text{ years}}$

ii) $d^{(12)} = 0.1 = 10\%$

$P.V = A.V \times \left(1 - \dfrac{d^{(12)}}{12}\right)^{12 \times n}$

Put $A.V = 2P.V$ in eqn.

$\therefore \cancel{P.V} = 2\cancel{PV} \times \left(1 - \dfrac{0.1}{12}\right)^{12n}$

$0.5 = (0.99166667)^{12n}$

taking ln on both sides

$\ln 0.5 = 12n \times \ln 0.9916667$

$n = \dfrac{\ln 0.5}{12 \ln 0.9916667}$

$= \boxed{6.90255067 \text{ years}}$