

Assignment Probability & Statistics

Q1

i) 4, 7, 9, 9, 11, 15, 18, 18, 18, 25

$$\text{Mean} = \frac{\sum x_i}{n} = \frac{134}{10} = 13.4$$

$$\text{Median} = \frac{n^{\text{th}} + n+1^{\text{th}}}{2} = \frac{10}{2} = 5 \Rightarrow \frac{5^{\text{th}} + 6^{\text{th}}}{2} = \frac{11 + 15}{2} = 13$$

$$\text{Mode} = 18$$

$$\text{Interquartile range} = Q_3 - Q_1 = \frac{9+7}{2} = 8, Q_3 = 18$$

$$\text{IQR} = Q_3 - Q_1 = 18 - 8 = 10$$

$$\text{Standard deviation} = -9.4, -6.4, -4.4, -4.4, -2.4, 1.6, 4.6, 4.6, 4.6, 11.6$$

$$= \frac{88.36 + 40.96 + 19.36 + 19.36 + 5.76 + 2.56 + 21.16 + 21.16 + 21.16 + 134.56}{10}$$

$$= \frac{374.4}{10} = 37.44$$

$$\text{Standard deviation} = \sqrt{37.44} = 6.19$$

x	0	1	2	3	4
f	5	11	26	15	3

Mean	x	f	f.x	f.x ²	cf
	0	5	0	0	5
	1	11	11	11	16
	2	26	52	104	42
	3	15	45	135	57
	4	3	12	48	60
		<u>60</u>	<u>120</u>	<u>298</u>	

$$\text{Mean} = \frac{\sum fx}{n} = \frac{120}{60} = 2$$

$$\text{Median} = \left(\frac{n+1}{2}\right)^{\text{th}} \text{ obs.} = \frac{61}{2} = 30.5$$

$$= 2$$

$$\text{Mode} = 2$$

$$\text{Standard Deviation} = \sqrt{\frac{\sum fx^2}{n} - \left(\frac{\sum fx}{n}\right)^2} = \sqrt{\frac{298}{60} - \left(\frac{120}{60}\right)^2} = \sqrt{\frac{58}{60}} = 0.9832$$

Interquartile range: $Q_1 = \left(\frac{n+1}{4}\right)^{\text{th}} \text{ observation} = \left(\frac{61}{4}\right)^{\text{th}} = (15.25)^{\text{th}} = 1$

$$Q_3 = \left(\frac{3(n+1)}{4}\right)^{\text{th}} = (45.75)^{\text{th}} = 3$$

$$IQR = Q_3 - Q_1 = 3 - 1 = 2$$

Q3 $\alpha = 2$ $\lambda = \frac{1}{2}$

$$\text{Mean} = \frac{\alpha}{\lambda} = \frac{2}{\frac{1}{2}} = 4$$

$$\text{standard dev.} = \frac{\sqrt{\alpha}}{\lambda} = 2\sqrt{2}$$

Q50

Sol $P(\text{late package sent via level 2}) = 40\% \times 50\% = 0.4 \times 0.5 = 0.2$

Q6

$X \sim U(1, 2)$ and $Y = \frac{3}{6-X}$ PDF of $Y = ?$

$$f_X(x) = \frac{1}{2-1} = 1 \quad X = 6 - \frac{3}{Y} \quad \frac{dx}{dY} = -\frac{3}{Y^2}$$

$$\text{PDF of } Y = \frac{3}{Y^2}$$

Q2

Sol $E(X) = \sum x P(X) = 1 \cdot 0.05 + 2 \cdot 0.15 + 3 \cdot 0.35 + 4 \cdot 0.4 + 5 \cdot 0.05$
 $= 0.05 + 0.3 + 1.05 + 1.6 + 0.25$
 $= 3.25$

$$V(X) = \sum x^2 P(X) - \mu^2 = 1^2 \cdot 0.05 + 4 \cdot 0.15 + 9 \cdot 0.35 + 16 \cdot 0.4 + 25 \cdot 0.05$$

 $= 11.45 - 10.5625 = 0.8875$

Coefficient of skewness = $\frac{E[(X-\mu)^3]}{\text{var}(X)^{1.5}}$

Q13 $X \sim U(75, 120)$ $P(X < 100)$

$$f_X(100) = \frac{100 - 75}{120 - 75} = \frac{25}{45} = \frac{5}{9}$$

Q14 $X \sim \text{Gamma}(5, 0.4)$ $P(X < 22.89)$

$$X \sim \chi^2_{10}$$

$$\propto 2 \cdot 0.4 X \chi^2_{2 \times 5}$$

$$0.8 X \chi^2_{10}$$

$$P(\chi^2_{10} < 18) = 0.9450$$

$$P(\chi^2_{10} < 18.5) = \frac{0.9529}{0.0079}$$

$$\odot P(\chi^2_{10} < 18.312) = \frac{0.0079 \propto 0.312}{0.5} = 0.994296$$