Q.1. $d^{(4)} = 8\%$

i. $d = 1 - \left(1 - \frac{d^{(4)}}{4}\right)^4$

$= 1 - \left(\frac{1 - 0.08}{4}\right)^4$

$= 0.07763184$

$d = 7.763184\%$

$\delta = -\ln(1-d)$

$= -\ln(1 - 0.07763184)$

$= 0.080811$

$\delta = 8.0811\%$ //

ii. $i = ?$

$i = \frac{d}{1-d}$

$= \frac{0.07763184}{1 - 0.0763184}$

~~$= 0.08416618$~~

$= 0.084086104$

$i = 8.404610420\%$ //

iii. $d^{(12)} = 12\left[1-(1-d)^{1/12}\right]$

$= 12\left[1-(1-0.07763184)^{1/12}\right]$

$= 0.080539$

$\boxed{d^{(12)} = 8.0539\%}$ //

Q.2. $\delta(t) = 0.004t + 0.0002t^2$

i. $PV = 1000.\left[e^{-\int_0^{10}(0.004t+0.0002t^2)dt}\right]$

$= 1000.\left(\left[e^{-\left[000.002\times100+\frac{0.0002\times1000}{3}\right]_0^{10}}\right)$

$= 1000.\left[e^{-\left(000.002\times1000+\frac{0.002\times1000}{3}\right)}\right]$

$= 1000.\left[e^{-(0.2+0.0667)}\right]$

$= 1000 \times e^{-(0.2667)}$

$\boxed{PV = 765.90281}$ //

ii. $i = ?$

$i = e^{\int_0^{10}(0.004t+0.0002t^2)dt}$

$= e^{\left(0.002\times100+\frac{0.0002\times1000}{3}\right)}-1$

$= e^{(0.2+0.0667)}-1$

$\boxed{i = 0.30565}$ //

Q.3.i. ~~Let~~ Assume that a person borrows 1 at an effective rate of discount 'd'. Then in effect, the original principle is '1-d' and the amount of discount is 'd'. From basic defination of 'i' as the ratio of the amount of discount to the principle,

$$i = \frac{d}{1-d}$$

This formula expresses 'i' as a function of 'd'

$$i = \frac{d}{1-d}$$

$$i - id = d$$

$$d(1+i) = i$$

$$d = \frac{i}{1+i}$$

$\therefore$ Discounting factor $v = \frac{1}{1+i}$

$$\therefore d = i \times \frac{1}{1+i}$$

$$\boxed{d = iv}$$

ii. (a) $\frac{d^{(4)}}{4} = 2.25\%$

$$d = 1 - \left(1 - \frac{d^{(4)}}{4}\right)^4$$

$$= 1 - \left(1 - \frac{0.0225}{4}\right)^4$$

$$d = 0.022311$$

$$d^{(2)} = 2\left[1 - (1 - 0.022311)^{1/2}\right]$$

$$\boxed{d^{(2)} = 2.244\%} \; //$$

(b) $d^{(0.5)} = 5\%$

$$d = 1 - \left(1 - \frac{0.05}{0.5}\right)^{0.5}$$

$$d = 0.051317 \; //$$

$$d^{(2)} = 2\left[1 - (1 - 0.051317)^{1/2}\right]$$

$$\boxed{d^{(2)} = 5.19\%} \; //$$

Q.4(i) ~~Le~~ Assume that a person borrows 1 at an effective rate of discount 'd'. Then in effect, the original principle is '1-d' and the amount of discount is 'd'. From the basic definition of 'i' ~~&~~ as the ratio of the amount of discount to the principle,

$$i = \frac{d}{1-d}$$

This ~~perform~~ formula express 'i' as an function of 'd'.

$$i = \frac{d}{1-d}$$

$$i - id = d$$

$$d(1+i) = i.$$

$$d = \frac{i}{1+i}$$

$$\therefore \text{Discounting factor } v = \frac{1}{1+i}$$

$$\therefore d = i \times \frac{1}{1+i}$$

$$\boxed{d = iv} \text{ //}$$

ii] $i = 0.08$

a) $d = \frac{0.08}{1+0.08}$

$d = 0.0740741$ //

$\boxed{d^{(12)} = 7.6715}$ //

(b) $i^{(365)} = 365\left[(1+0.08)^{1/365} - 1\right]$

$\boxed{i^{365} = 7.69692\%}$ //

(c) $\delta = \ln(1+i)$

$= \ln(1+1.08)$

$\boxed{\delta = 7.6961\%}$ //

(d) $i^{(0.5)} = 0.5\left[(1+0.08)^{1/0.5} - 1\right]$

$\boxed{i^{(0.5)} = 8.32\%}$ //

5. (i) $d^{(6)} = 6\%$

a.) $i^{(2)} = ?$

$d = 1 - \left(1 - \frac{d^{(4)}}{4}\right)^4$

$= 1 - \left(1 - \frac{0.06}{4}\right)^4$

$$d = 5.8663\%$$

$$i = \frac{0.058663}{1 - 0.058663}$$

$$i = 6.232\%$$

$$i^{(2)} = 2\left[(1 + 0.06323)^{1/2} - 1\right]$$

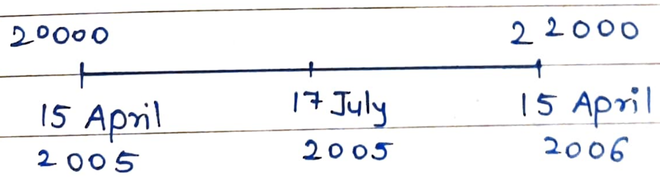
$$i^{(2)} = 6.138\% \ //$$

b) $d = 5.8663\%$

$$d^{(12)} = 12\left[1 - (1 - 0.058663)^{1/12}\right]$$

$$d^{(12)} = 6.0302\% \ //$$

ii]



Loan paid in 93 days.

$$93 \text{ days} = 20000 + 20000\left(\frac{93}{365}\right)$$

$$93 \text{ days} = 205095.89$$

6. ~~1x~~ Assume cash payment of 100

$\therefore$ Discounted value of 100 at time 0 $= 100 \times \frac{70}{100}$

$= 70$

At six month $= 100 \times 25\%$.
$= 75$

$$PV_0 = AV(1-d)^n$$

$$70 = 75(1-d)^{1/12}$$

$$\left(\frac{70}{75}\right)^2 = 1-d$$

$$d = 0.1288889$$

$$d = 12.8889\%$$

$$d^{(12)} = 12\left[1-(1-0.1288889)^{1/12}\right]$$

$$= 13.71956\%$$

$$i^{(2)} = ?$$

$$i = \frac{0.1288889}{1-0.1288889}$$

$$= 0.1479593.$$

$$i^{(2)} = 2\left[(1+0.1479593)^{1/2} - 1\right]$$

$$= 14.2857\%$$

$$\delta = -\ln(1-d)$$

$$\delta = 13.7986\% \quad //$$

7.(i) $PV_0 = 20000$

$AV_{17} = 26000$

$$20000 = 26000\left(1+\frac{i^{(4)}}{4}\right)^{4\times\frac{17}{12}}$$

$$\left(\frac{20000}{26000}\right)^{3/17} = 1+\frac{i^{(4)}}{4}$$

$$i^{(4)} = 18.96\% \quad //$$

ii) $d^{(2)} = ?$

$$i = \left(1+\frac{0.1896}{4}\right)^{4} - 1$$

$$= 20.34541$$

$$d = \frac{0.203451}{1+0.2034541}$$

$$= 16.90558867$$

$$d^{(2)} = 2\left[1-(1-0.1690558867)^{1/2}\right]$$

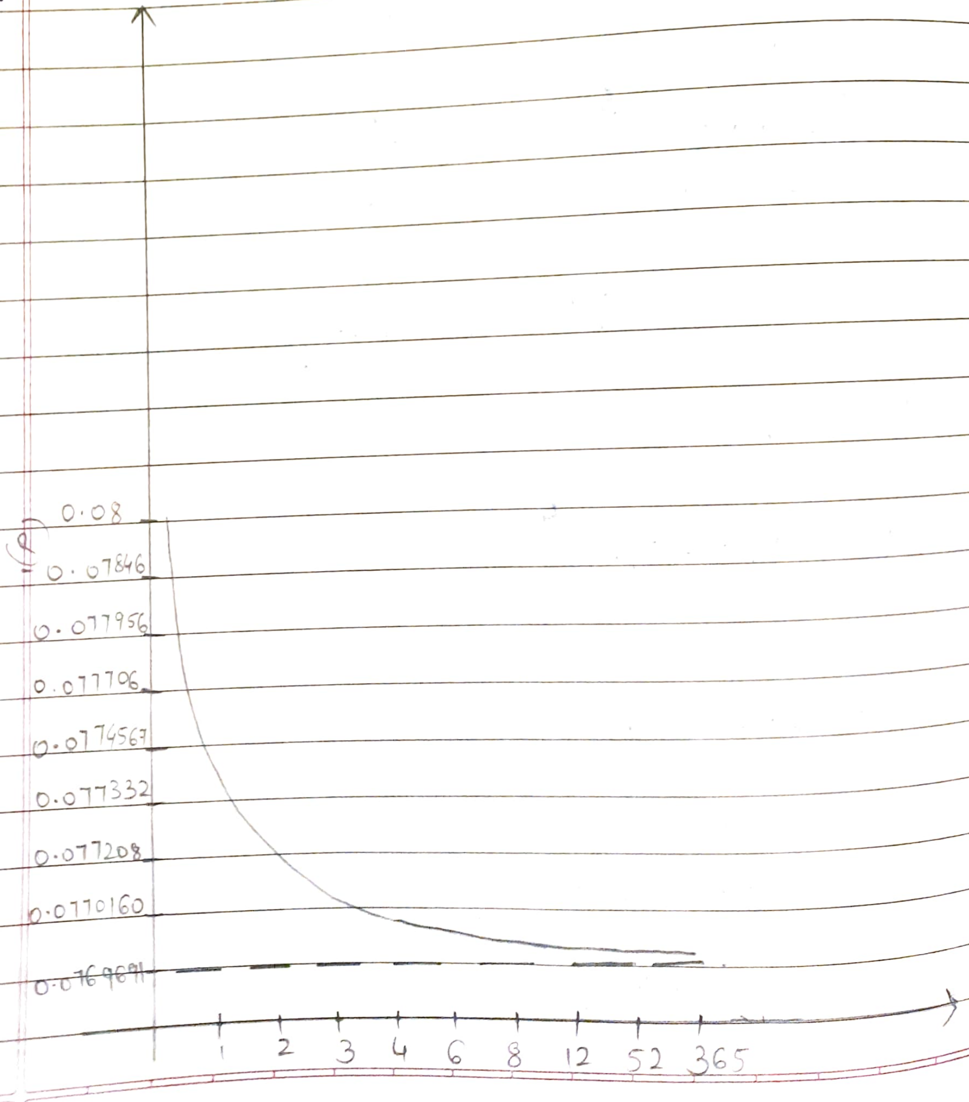
$$= 3.062850784\% \;/\!/$$

iii] $$26000 = 20000(1+i)^{17/12}$$

$$\left(\frac{26000}{20000}\right)^{12/17} = 1+i$$

$$i = 21.8\% \;/\!/$$

Q.8.



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9.i. Force of interest ~~rfer~~ refers to nominal interest rate or a discount rate compounded infinite no. of times continously per time period.

ii. $i^{(2)} = 0.0775$

$$i = \left(1 + \frac{i^m}{m}\right) - 1$$

$$= \left(1 + \frac{0.07775}{2}\right)^2 - 1$$

$i = 7.9002\%$

$$\delta = \ln(1+i)$$
$$= \ln(1 + 0.079002)$$
$$= \ln(1.079002)$$
$$\delta = 0.07604$$

$d^{(12)} = ?$

$$d = \frac{0.079002}{1.079002}$$

$$= 0.073322$$

$$d^{(12)} = 12\left[1 - (1 - 0.07322)^{1/12}\right]$$

$$= 0.07599$$

$d^{(12)} = 7.599\%$

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$$i^{(12)} = \frac{1}{2} \left[(1+i)^{1/12} - 1\right]$$

$$i^{(12)} = 8.2123\% \,//$$

Q.10. $\cancel{PV_0 = C \times V(t)}$

$$PV_0 = C \times V(t)$$
$$= 1 \times V(t)$$
$$= V(t)$$

$$V(t) = e^{-\int_0^5 \delta(5)\, dt}$$

$$= e^{-\int_0^5 0.08\, dt} \cdot e^{-\int_0^{10} 0.06 dt \, - \, \int_{10}^{t} 0.04 dt}$$

$$= e^{-(0.08 \times 5)} \cdot e^{-(0.06 \times 10^{e} - 0.06 \times 5) - (0.04t - 0.3)}$$

$$= e^{-0.4} \cdot e^{-0.3^{e}} - (0.04t - 0.4)^{\cdot e}$$

$$= e^{-0.4 - 0.3 - 0.04t + 0.4}$$

$$v(t) = e^{-0.3 - 0.04t}. \quad //$$

Q.11.(a) $AV_{9/12} = 50000$

$$d = 18\%.$$

$$PV = 50000 \, (1 - 0.18)^{1/12}$$

$$PV = 43085.43 \,//$$

b. $\delta = 6\%$

$d = 1 - e^{-\delta}$

$d = 5.82355\%$

$d^{(12)} = 12\left[1 - (1 - 0.0582355)^{1/12}\right]$

$d^{(12)} = 5.99\%$

ii. $d^{(4)} = 6\%$

$i^{(2)} = ?$

$d = 1 - \left(1 - \frac{0.06}{4}\right)^4$

$= 5.866345\%$

$i = \frac{0.05866345}{1 - 0.05866345}$

$= 6.23\%$

$i^{(12)} = 12\left[(1 + 0.0623)^{1/12} - 1\right]$

$= 6.0589\%$

$i^{(12)} = 6.059\%$ //

(c) $d < \delta < i^{(4)} < i$

(d) ~~As~~

Q.12. $PV_0 = 7049$

$i^{(2)} = 6\%$

$i^{(4)} = 1.5\%$

$d^{(12)} = 1.5\%$

$d^{(12)} = 6\%$

$$d = 1 - \left(1 - \frac{d^{(12)}}{12}\right)^{12}$$

$= 0.0584$

$$AV_6 = 7049\left[\left(1 + \frac{i^{12}}{12}\right)^{12\times2} \cdot \left(1 + \frac{i^4}{4}\right)^{4\times\frac{30}{12}} \cdot (1-d)^{18/12}\right]$$

$$= 7049\left[\left(1 + \frac{0.06}{12}\right)^{24} \cdot \left(1 + \frac{0.015}{4}\right)^{10} \cdot (1 - 0.0584)^{2/3}\right]$$

$= 7049\,[(1.127150) \times (1.03814) \times (0.913692)]$

$AV_6 = 7536.4154$ //

Q.13 (i) $(1+i)^n = 2$

$(1+i)^n = 2$

$$\boxed{n = 7.3 \text{ years}}$$

(ii) $d^{(12)} = 10\%$.

$$d = 1 - \left[1 - \frac{d^{(12)}}{12}\right]^{12}$$

$d = 9.55\%$.

$x = 2x(1-d)^n$

$\frac{1}{2} = (1-d)^n$

$$\frac{\ln(0.5)}{\ln(0.9045)} = n$$

$$\boxed{n = 6.9 \text{ years.}}$$