NMA assignment - 1

Q1) Measured radius = 12.65
Actual radius = 12.5

i) Absolute error = 12.65 - 12.5 = 0.15

ii) Relative error = $\frac{(12.65 - 12.5)}{(12.5)}$ = 0.012

iii) Percentage error = $\frac{(12.65 - 12.5)}{(12.5)} \times 100$ = 1.2%

Q5) $A^2 = A$

$7A - (I + A)^3$

$7A - (I^3 + 3I^2A + 3IA^2 + A^3)$

$7A - (I + 3A + 3A + A \cdot A^2)$

$7A - I - 6A - A \cdot A$

$A - I - A^2$

$A - I - A$

$= -I$

Q6 $\begin{bmatrix} 1 & -1 & 2 \\ 4 & 0 & 6 \\ 0 & 1 & -1 \end{bmatrix} = A.$

$|A| = 1(0-6) + 1(-4-0) + 2(4-0)$

$= -6 - 4 + 8$

$= -2$

Cofactors

$A_{11} = (-6)(1) = -6$

$A_{12} = (-4)(1) = 4$

$A_{13} = (4)(1) = 4$

$A_{21} = (-1)(-1) = 1$

$A_{22} = (-3)(1) = -3$

$A_{23} = (2)(-1) = -2$

$A_{31} = (-8)(1) = -8$

$A_{32} = (-2)(-1) = 2$

$A_{33} = (4)(1) = 4$

Cofactor Matrix $= \begin{bmatrix} -6 & 4 & 4 \\ 1 & -3 & -2 \\ -8 & 2 & 4 \end{bmatrix}$

Adj $A = \begin{bmatrix} -6 & 1 & -8 \\ 4 & -3 & 2 \\ 4 & -2 & 4 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} \text{Adj } A.$

$= \frac{1}{-2} \begin{bmatrix} -6 & 1 & -8 \\ 4 & -3 & 2 \\ 4 & -2 & 4 \end{bmatrix}$

$= \frac{1}{2} \begin{bmatrix} 6 & -1 & 8 \\ -4 & 3 & -2 \\ -4 & 2 & -4 \end{bmatrix}$ //

Q10) $t_6 = ar^5 = 32$

$t_8 = ar^7 = 128$

$\frac{t_8}{t_6} = \frac{ar^7}{ar^5} = \frac{128}{32}$

$r^2 = 4$

$r = 2$

Q11) $(4+3x)^5$

$= {}^5C_0 \cdot 4^5 + {}^5C_1 \cdot (4)^4 \cdot (3x) + {}^5C_2 \cdot (4)^3 \cdot (3x)^2 + {}^5C_3 \cdot (4)^2 \cdot (3x)^3$
$+ {}^5C_4 \cdot 4 \cdot (3x)^4 + {}^5C_5 \cdot (3x)^5$

$= 1024 + 5(256)(3x) + 10(64)\cdot(9x^2) + 10(16)\cdot(27x^3)$
$+ 5(4)(81x^4) + 243x^5$

$= 1024 + 3840x + 5760x^2 + 4320x^3 + 1620x^4 + 243x^5$

Q12) $\begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix} = A$

$A - \lambda I = \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$

$= \begin{bmatrix} 2-\lambda & -3 & 0 \\ 2 & -5-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{bmatrix}$

$|A - \lambda I| = 0$

$$\begin{vmatrix} 2-\lambda & -3 & 0 \\ 2 & -5-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{vmatrix} = 0$$

$$(2-\lambda)[(-5-\lambda)(3-\lambda)] + 3[2(3-\lambda)] + 0 = 0$$

$$-(2-\lambda)(5+\lambda)(3-\lambda) + 6(3-\lambda) = 0$$

$$(3-\lambda)[(2-\lambda)(5+\lambda) + 6] = 0$$

$$-(3-\lambda)[10 + 2\lambda - 5\lambda - \lambda^2 + 6] = 0$$

$$-(3-\lambda)[-\lambda^2 - 3\lambda + 16] = 0$$

$$(3-\lambda)(\lambda^2 + 3\lambda - 16) = 0$$

$$3-\lambda = 0$$

$$\lambda = 3 \text{ or}$$

$$\lambda^2 + 3\lambda - 16 = 0.$$

$$\therefore \lambda = \frac{-3 \pm \sqrt{3^2 - 4(1)(-16)}}{2}$$

$$\lambda = \frac{-3 \pm \sqrt{73}}{2}$$

$$\therefore \lambda = 3 \text{ or } \frac{-3 + \sqrt{73}}{2} \text{ or } \frac{-3 - \sqrt{73}}{2}$$

14) $(1 - x + x^2)^4$

$(x^2 - x + 1)^4$

let $x^2 - x$ be a

$(a + 1)^4$

$^4C_0 a^4 + {}^4C_1 a^3 + {}^4C_2 a^2 + {}^4C_3 a + {}^4C_4 \cdot 1$

$a^4 + 4a^3 + 6a^2 + 4a + 1$

$(x^2 - x)^4 + 4(x^2 - x)^3 + 6(x^2 - x)^2 + 4(x^2 - x) + 1$

$(x^2)^4 + 4(x^2)^3 \cdot x + 6(x^2)^2 x^2 + 4x^2 \cdot x$ ~~x~~ ~~1~~ =

2)

x	$\log(x)$
4.0	0.60206
4.5	0.6532125
5.5	0.7403627
6	0.7781513.

i) $\log(5) = \log(4) + \frac{1}{2}(\log(6) - \log(4))$

$= 0.60206 + \frac{1}{2}(0.7781513 - 0.60206)$

$= 0.69905$ //

ii) $\log(5) = 0.6532125 + \frac{1}{2}(0.7403627 - 0.6532125)$

$= 0.6967876$ //

Q3) $x^4 - x - 1 = 0$

a = 1.5

b = 2

Iterations = 5

x	y
1.5	-6.4375
2	4

$x_1 = \frac{1.5+2}{2} = 1.75$

$y_1 = -2.37109375$

$x_2 = \frac{1.75+2}{2} = 1.875$

$y_2 = 0.4846131$

$x_3 = \frac{1.75+1.875}{2} = 1.8125$

$y_3 = -1.026248431$

$x_4 = \frac{1.875+1.8125}{2} = 1.84375$

$y_4 = -0.287734$

$x_5 = \frac{1.84375+1.875}{2} = 1.859375$

$y_5 = 0.09337812$

Q4) $f(x) = x^3 - x - 1$

x	y
0	-1
1	-1
2	5

$f(x) = x^3 - x - 1$

$f'(x) = 3x^2 - 1$

$x_0 = 1$

$f(x_0) = -1$

$f'(x_0) = 2.$

$x_1 = 1 + 1/2 = 1.5$

$f(x_1) = 0.875$

$f'(x_1) = 5.75$

$x_2 = 1.5 - \frac{0.875}{5.75} = 1.3478$

$f(x_2) = 0.1006.$

$f'(x_2) = 4.4497.$

$x_3 = 1.3478 - \frac{0.1006}{4.4497} = 1.3252$

$f(x_3) = 0.0019$

$f'(x_3) = 4.2684.$

$x_4 = 1.3252 - \frac{0.0019}{4.2684} = 1.3247$

$f(x_4) = -0.000078.$

$\therefore$ x is approximately 1.3247

(Q9) $a = 101$

$a_n = 998$

$d = 3$

$a_n = a + (n-1)d$

$998 = 101 + (n-1)3$

$3(n-1) = 998 - 101$

$(n-1) = \frac{897}{3} = 299$

$n = 300$

$S_n = \frac{300}{2}(101 + 998)$

$= 150 \times 1099$

$= 164850$

(Q15) $e^{-x} = 3\log(x)$

$e^{-x} - 3\log(x) = 0$

$a = 1$

$b = 2$

Iterations: 4

	x	y
1		0.367879412
2		-0.767754703 8

$x_1 = \frac{1+2}{2} = 1.5$

$y_1 = -0.305143617$

$x_2 = \frac{1+1.5}{2} = 1.25$

$y_2 = -0.000422524$

$x_3 = \frac{1+1.25}{2} = 1.125$

$y_3 = 0.171194 9$

$x_4 = \frac{1.125+1.25}{2} = 1.1875$

$y_4 = 0.081081382$

13) $f(x) = x^3 - 12^2 + 8x - 3$

$f'(x) = 3x^2 - 144x + 8$

$x_0 = 5$

$f(x_0) = -13$

$f'(x_0) = 18$

$x_1 = 5 + \frac{13}{13} = 6$

$f(x_1) = 9$

$f'(x_1) = 32.$

$x_2 = 6 - \frac{9}{32} = 5.71875$

$f(x_2) = 0.8478698$

Q8) $Mag = \sqrt{(75)^2 + (75)^2}$

$= \sqrt{5625 + 625}$

$= \sqrt{6250}$

$= 79.057$

20) ~~[illegible]~~

Q7 $A = 8\hat{i} - 9\hat{j}$

$B = 7\hat{i} - 9\hat{j}$

$|A| = \sqrt{8^2 + 9^2} = \sqrt{145}$

$|B| = \sqrt{7^2 + 9^2} = \sqrt{130}$

$A \cdot B = (8\hat{i} - 9\hat{j}) \cdot (7\hat{i} - 9\hat{j})$

$= 8 \times 7 + (-9 \times -9)$

$= 56 + 81$

$= 137$

$A \cdot B = |A| \cdot |B| \cdot \cos\theta$

$137 = \sqrt{145} \times \sqrt{130} \cdot \cos\theta$

$137 = 5\sqrt{754} \times \cos\theta$

$\cos\theta = \frac{137}{5\sqrt{754}}$

$\cos\theta = 0.997849$

$\theta = \cos^{-1}(0.997849)$

$= 3.758683119$