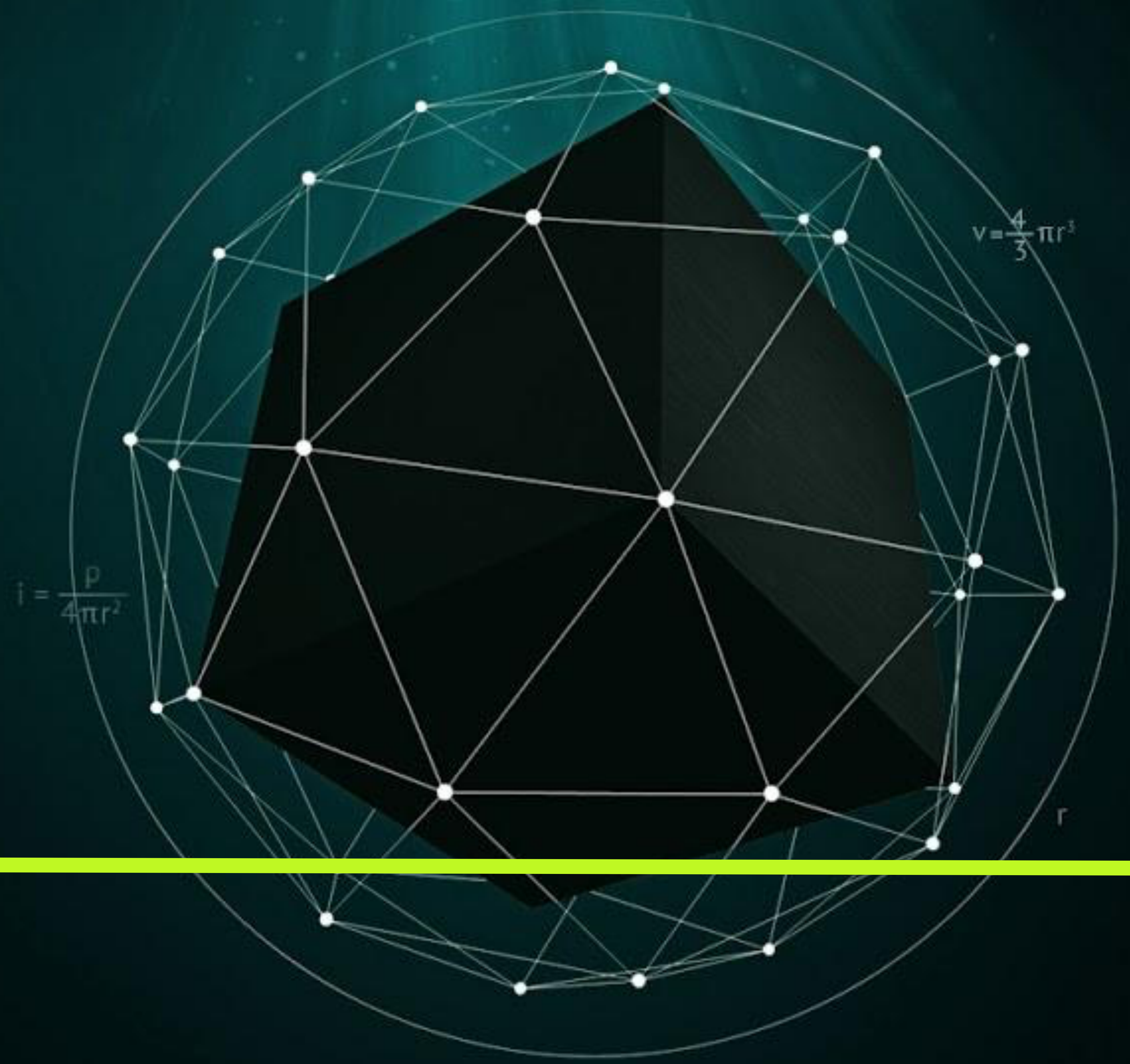
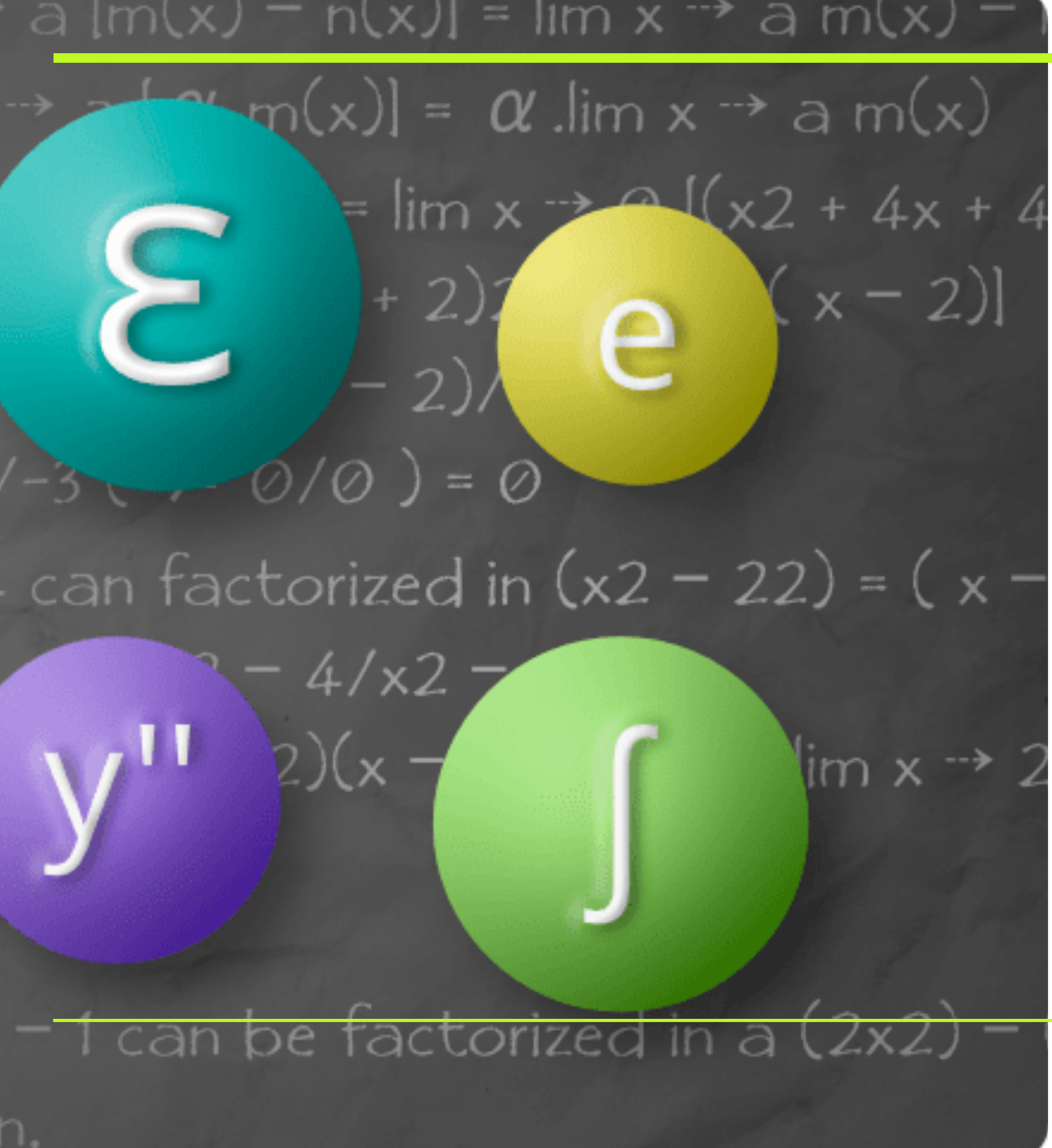
Use of calculus in calculating time of death of a dead body during post-mortem



CALCULUS

-GROUP 1



What is calculus?

- Calculus is a branch of mathematics focused on limits, functions, derivatives, integrals, and infinite series. This subject constitutes a major part of contemporary mathematics education. Calculus has widespread applications in science, economics, and engineering and can solve many problems for which algebra alone is insufficient.

Branches of calculus:

- Differential calculus
- Integral calculus
- Multivariable calculus
- Fractional calculus
- Differential Geometry

History of calculus

- Calculus, known in its early history as infinitesimal calculus, is a mathematical discipline focused on limits, continuity, derivatives, integrals, and infinite series.
- Isaac Newton and Gottfried Wilhelm Leibniz independently developed the theory of infinitesimal calculus in the later 17th century.
- By the end of the 17th century, both Leibniz and Newton claimed that the other had stolen his work, and the Leibniz–Newton calculus controversy continued until the death of Leibniz in 1716.

Calculus scholars

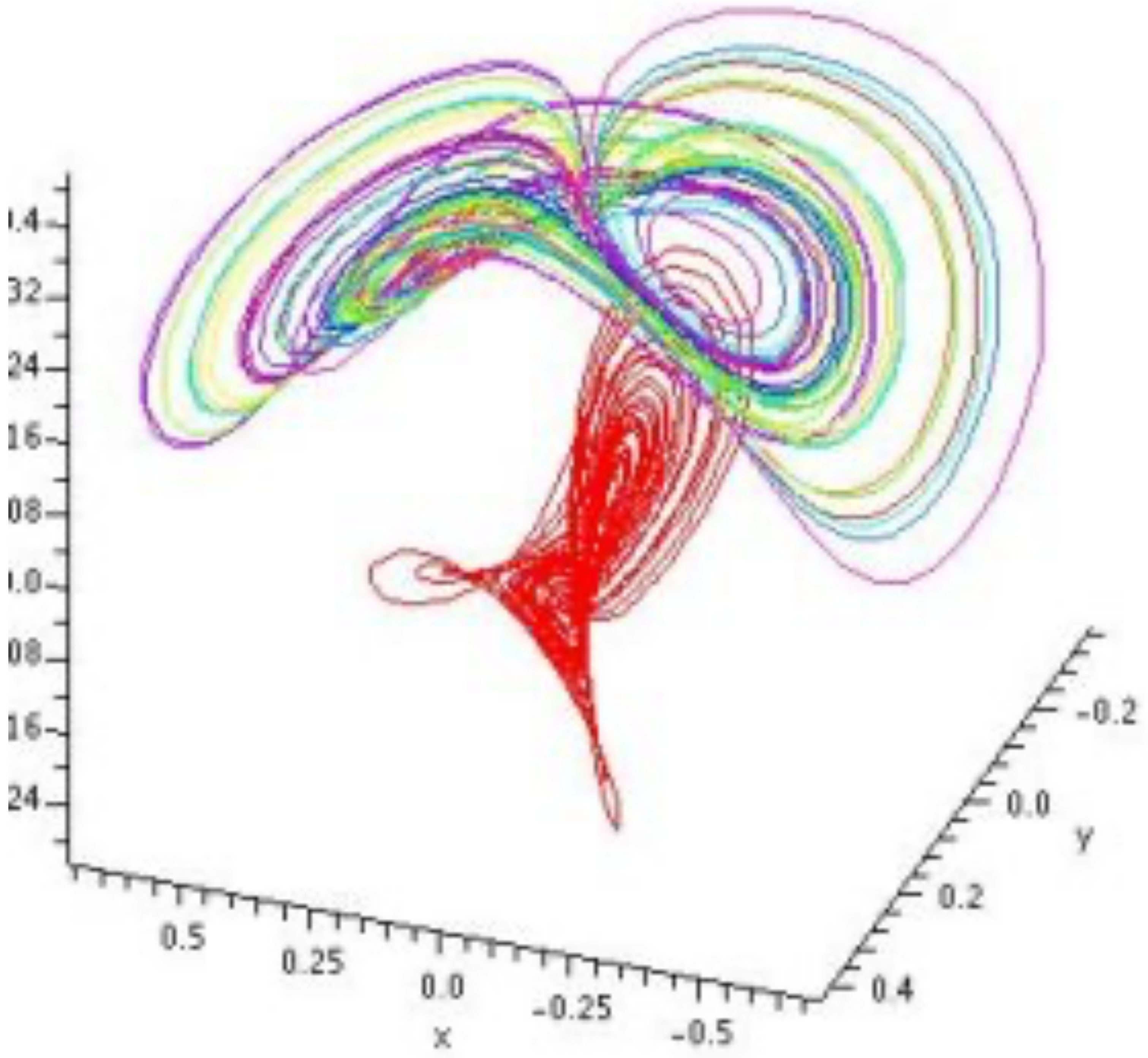
- Sir Isaac Newton
- Gottfried Leibniz

Differential equations:

It is an equation which consists of one or more functions with its derivatives. A differential equation contains derivatives which are either partial derivatives or ordinary derivatives. The derivative represents a rate of change, and the differential equation describes a relationship between the quantity that is continuously varying with respect to the change in another quantity. It is mainly used in fields such as physics, engineering, biology and so on.

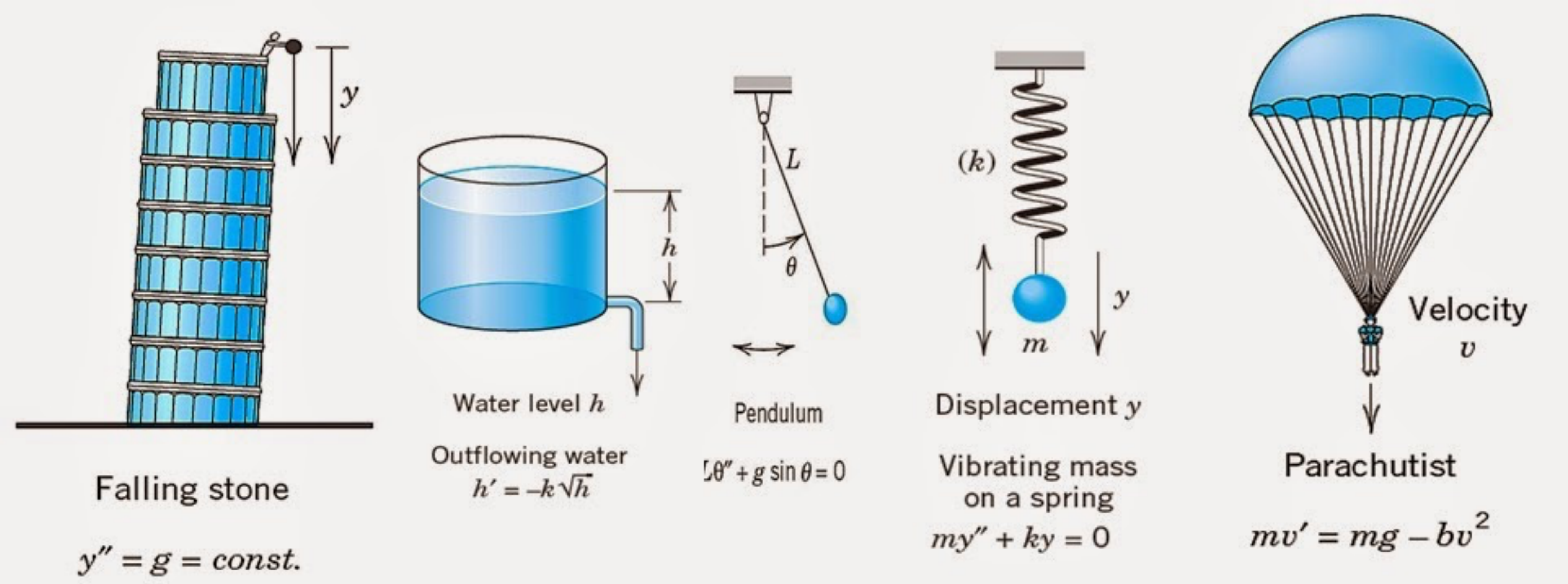
Types of differential equations:

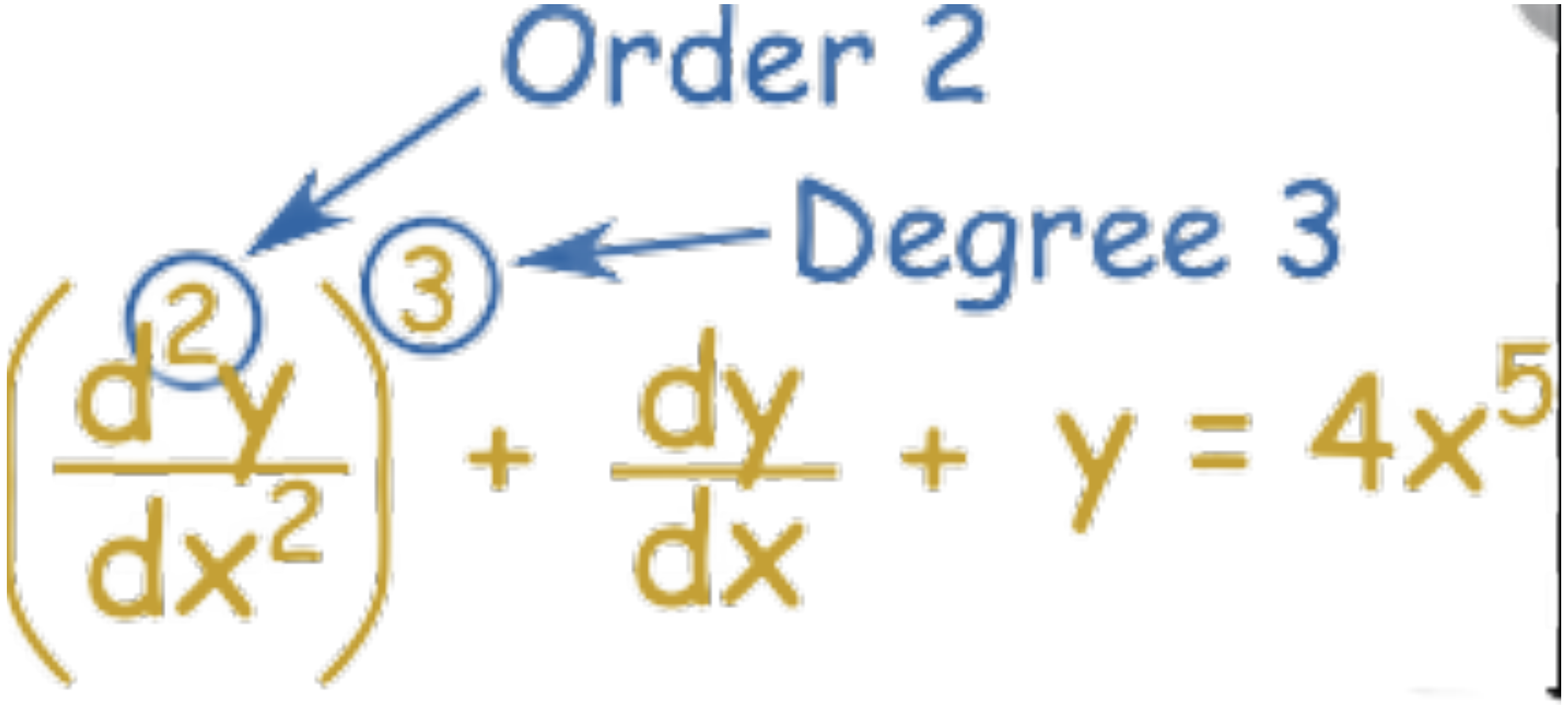
- Ordinary Differential Equations
- Partial Differential Equations
- Linear Differential Equations
- Nonlinear differential equations
- Homogeneous Differential Equations
- Nonhomogeneous Differential Equations



- How are they useful?

In our world things change, and describing how they change often ends up as a Differential Equation. Differential Equations can describe how populations change, how heat moves, how springs vibrate, how radioactive material decays and much more. They are a very natural way to describe many things in the universe.





- What to do with them?

On its own, a Differential Equation is a wonderful way to express something, but is hard to use. We try to solve them by turning the Differential Equation into a simpler equation without the differential bits, so we can do calculations, make graphs, predict the future, and so on.

Applications of differential equations:

Differential equations have several applications in different fields such as applied mathematics, science, and engineering. Apart from the technical applications, they are also used in solving many real-life problems:

1) Differential equations describe various exponential growths and decays.

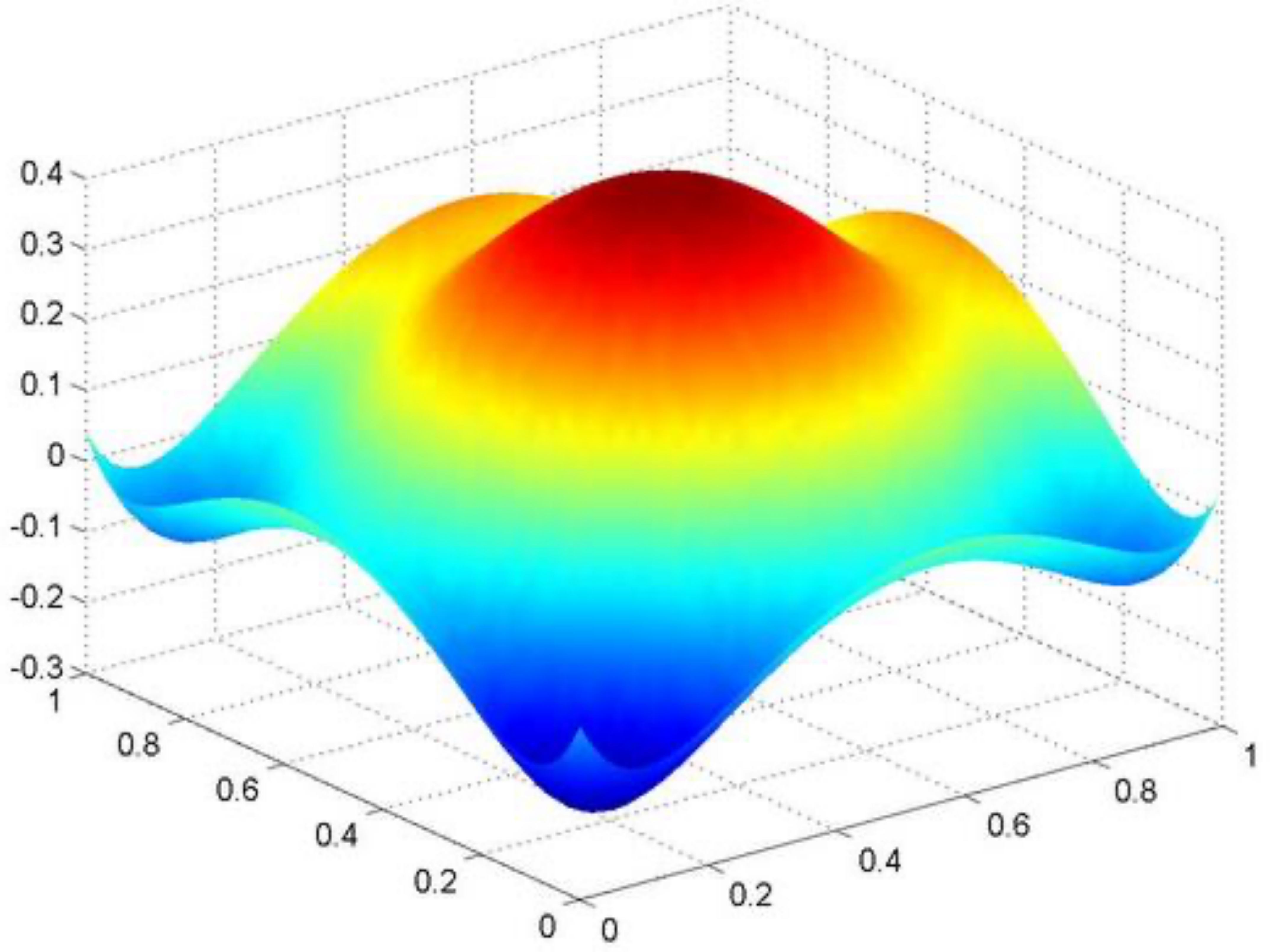
2) They are also used to describe the change in return on investment over time.

3) They are used in the field of medical science for modelling cancer growth or the spread of disease in the body.

4) Movement of electricity can also be described with the help of it.

5) They help economists in finding optimum investment strategies.

The various other applications in engineering are: heat conduction analysis, in physics it can be used to understand the motion of waves. The ordinary differential equation can be utilized as an application in the engineering field for finding the relationship between various parts of the bridge.



Use of calculus :

In calculating time of death of a dead body during post-mortem

A post mortem examination is a medical examination carried out on the body after death.

The Factors in Estimating Time of Death

- Rigor Mortis

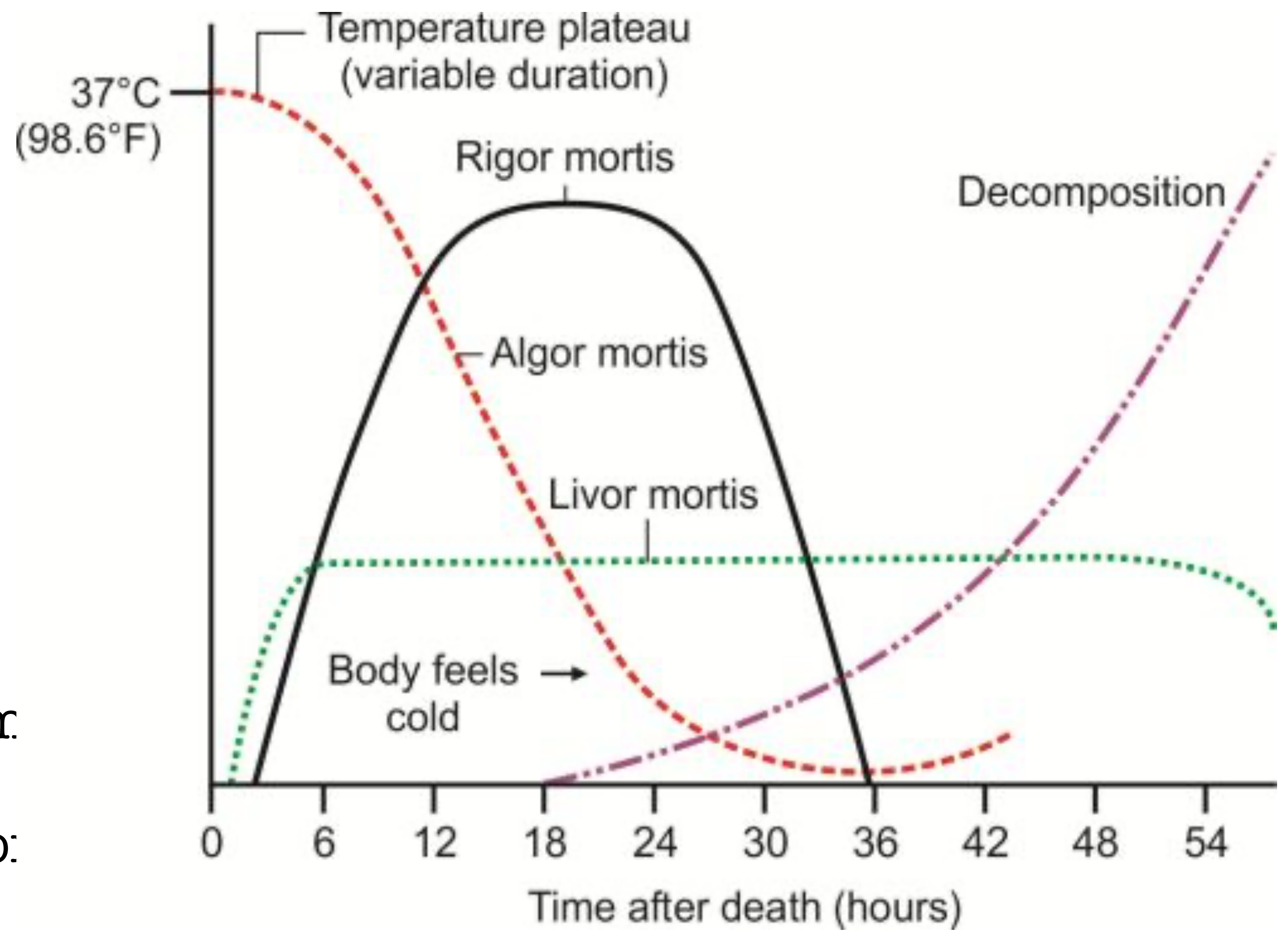
The gradual muscle stiffening that spreads over the body in the hours after death then fades as gradual as it started.

- Algor Mortis

The slow cooling of a warmblooded corpse as it equilibrates with the temperature or its surroundings.

- Livor Mortis

The dark blue or red discoloration of the skin caused by the pooling of blood due to gravity. This begins the moment blood pressure plummets to zero.Anywhere from 24 to 48 hours after death, lividity reaches its peak, The body reaches room temperature, and rigor mortis disappears. Therefore, estimating the time of death is more accurate within the 24 hours after death. Anywhere from 2 days or more these factors are no longer valuable in the investigation.



Other Methods:

Glaister equation:

Algor mortis, though useful to calculate PMI, isn't always the most reliable factor. The Glaister equation (also called the 'rule of thumb') is a general formula used to back-calculate the rate of cooling (when the ambient temperature is less than the body temperature).

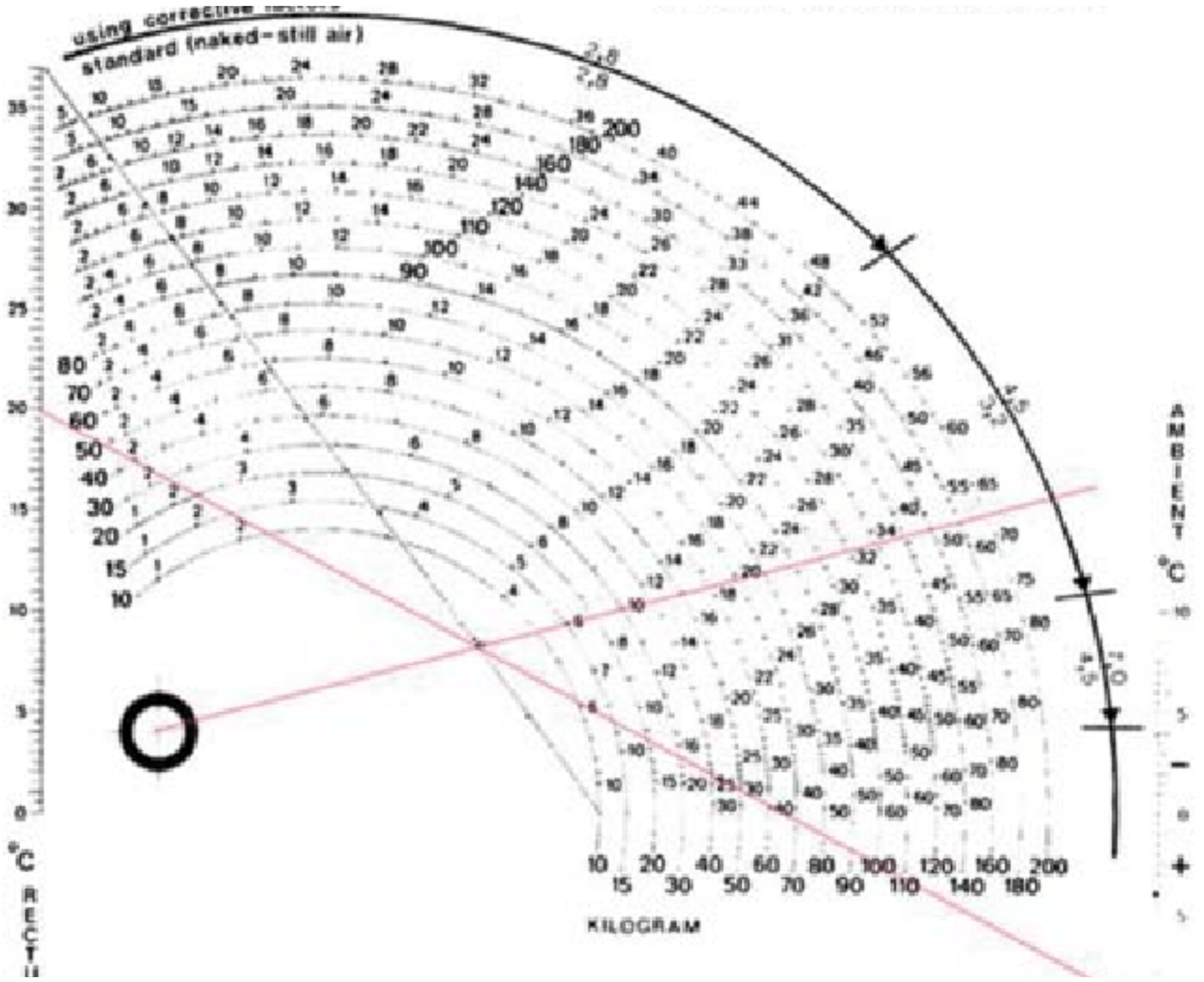
$$(36.9^\circ C - \text{rectal temperature in Celsius}) \cdot \frac{6}{5}$$

or

$$\frac{98.4\,^\circ\text{F} - \text{rectal temperature in Fahrenheit}}{1.5}$$

Henssge Nomogram Technique

The Henssge Nomogram Technique is another method of using Algor mortis to calculate PMI. A nomogram is a graphical calculating device. There is a graph with different parameters, and matching one or more known parameters allows one to estimate a third unknown parameter using the graph. The Henssge nomogram takes into account the body weight of the individual and the rectal temperature to estimate the PMI.



Newton's Law of Cooling

"The rate of cooling of a body is directly proportional to the difference in temperatures of the body (T) and the surrounding (T_0), provided difference in temperature should not exceed by 30^{0}C."

In terms of temperature differences, it results in a simple differential equation expressing temperature-difference as a function of time, the solution to which describes an exponential decrease of the same. This characteristic decay of the temperature-difference is also associated with Newton's law of cooling.

Limitations of Newton's Law of Cooling

- The difference between the temperatures must be small.
- The loss of heat from the body should be by radiation only.
- The temperature of surroundings must remain constant while cooling of the body.

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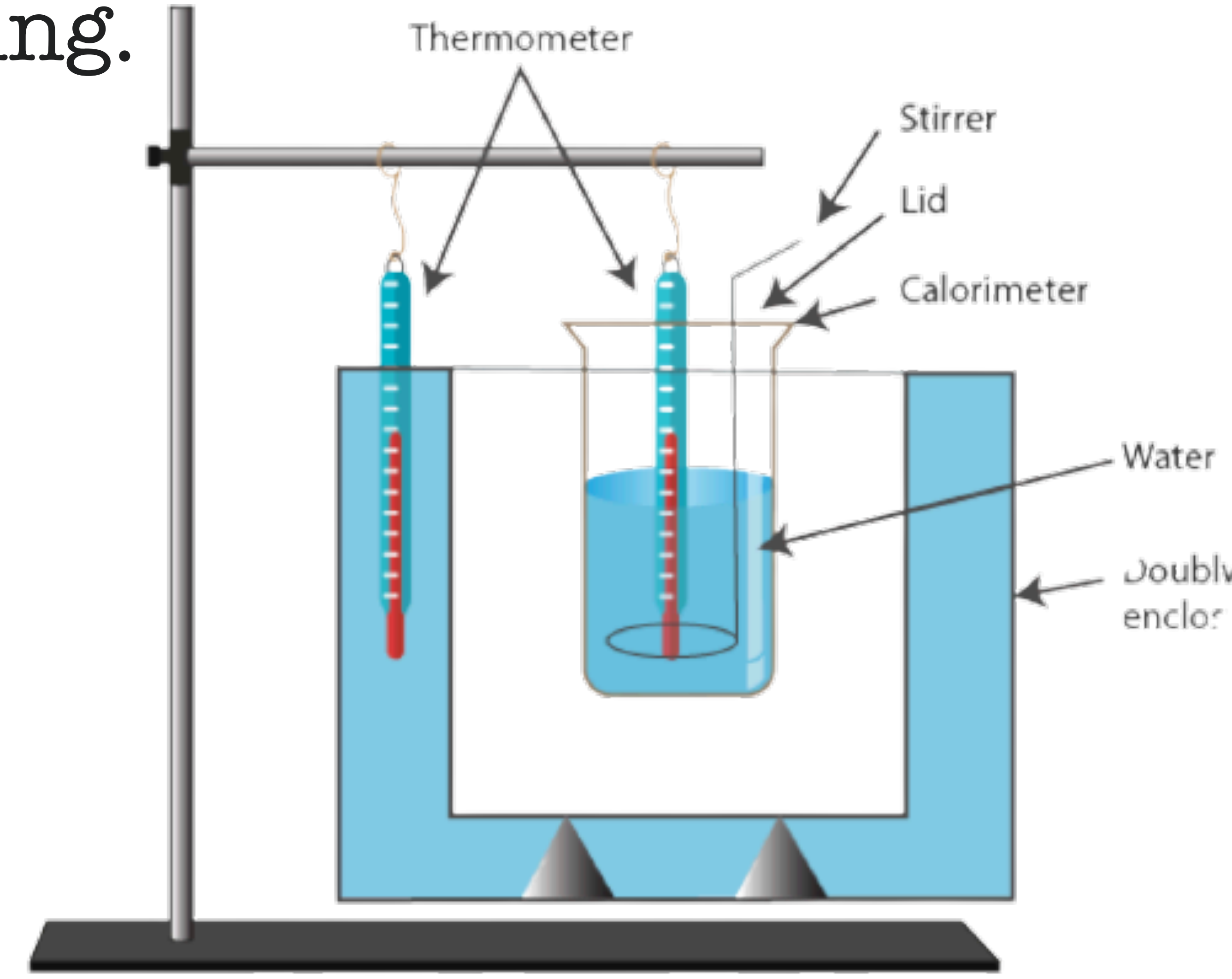


Newton's law of cooling appartus

MATHEMATICAL FORMULATION OF NEWTONS LAW

$$Q = hA\left(T(t) - T_{env}\right) = hA \triangle T(t)$$

where

- Q is the rate of heat transfer out of the body (SI unit: watt)
- h is the heat transfer coefficient (SI unit: W/m^2 . K)
- A is the heat transfer surface area (SI unit: m^2)
- T is the temperature of the object's surface (SI unit: K)
- T_{env} is the temperature of the environment (SI unit: K)
- $\triangle T(t) = T(t) - T_{env}$ is the time-dependent temperature difference between environment and object (SI unit: K)

Newton's Law of Cooling Derivation

For small temperature difference between a body and its surrounding, the rate of cooling of the body is directly proportional to the temperature difference and the surface area exposed.

$dQ/dt \propto (q - q_s)]$, where q and q_s are temperature corresponding to object and surroundings.

From above expression, $dQ/dt = -k[q - q_s)]$ . . . . (1)

This expression represents Newton's law of cooling. It can be derived directly from Stefans law which gives,

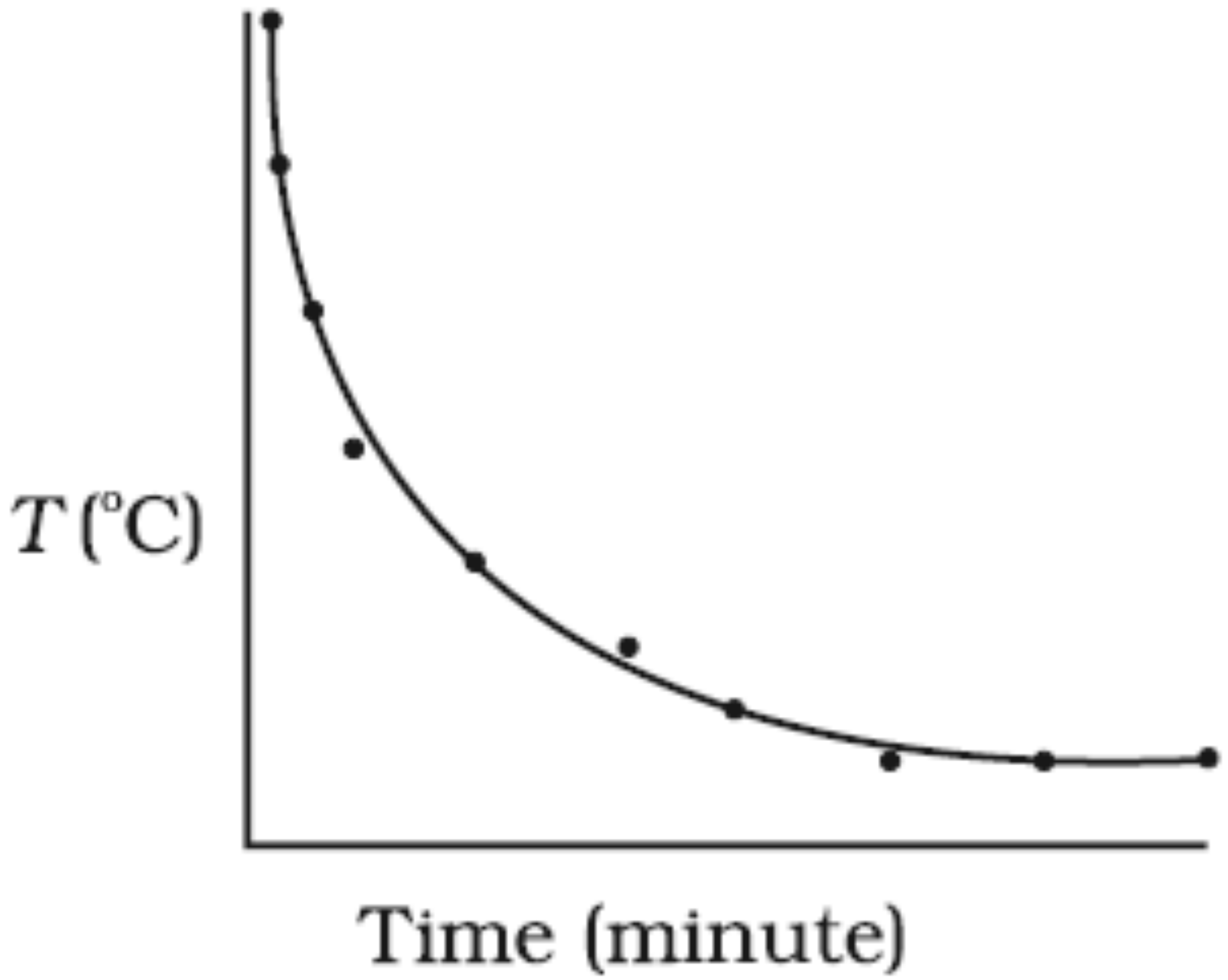
$k = [4e\sigma \times \theta^3{}_0/mc]\ A$ . . . . . (2)

Now, $d\theta/dt = -k[\theta - \theta_o]$

$$\int_{\theta_1}^{\theta_2} \frac{d\theta}{(\theta - \theta_\circ)} = \int_0^1 -k\,dt$$

where,

- q_i = initial temperature of object,
- q_f = final temperature of object.
- $\ln (q_f - q_0)/(q_i - q_0) = kt$
- $(q_f - q_0) = (q_i - q_0)\, e^{-kt}$
- $q_f = q_0 + (q_i - q_0)\, e^{-kt}$ . . . . . . (3).



Curve showing cooling of hot water with time.

Algor mortis with Newtons law of Cooling

Newton's Law of Cooling was originally applied to smaller masses with known starting temperatures, but after deriving another formula from the integrated original, one can apply Newton's Law with body temperature to estimate the body's cooling rate.

The derivation formula that is specific for estimating the time of death of a body is as stated below.

$$\boxed{t=-10\ln[(T_B-T_R)\div(98.6-T_R)]}$$

t= time in hours
T_B= Body temperature (°F)
T_R= Room temperature (°F)

Example:

The Case of the Murdered Accountant

After a busy evening of income calculations an accountant was found dead in his office.

In the Murder Investigation, the temperature of the corpse was $32.5^\circ C$ at 1:30 PM and $30.3^\circ C$ an hour later .The temperature of the surroundings was $20^\circ C$.

*Assuming the body temperature at death was 37 °C, what would the estimated time of death be to the nearest minute?

Solution:

Notice, according to newton's law of cooling, rate of cooling $\frac{dT}{dt}$ of a body is directly proportional to the temperature difference $(T - T_\infty)$ between system & surrounding

$$\frac{dT}{dt} = -k(T - T_\infty)$$

Where, T is the instantaneous temperature & T_∞ is surrounding temperature. The negative sign indicates the decrease in the temperature w.r.t. time t. Now we have

$$\frac{dT}{T - T_\infty} = -kdt$$

$$\int \frac{dT}{T - T_\infty} = -\int kdt$$

$$\ln(T - T_\infty) = -kt + C$$

Now, at time $t = 0$ the initial temperature is T_i then we have

$$\ln(T_i - T_\infty) = -k(0) + C \iff C = \ln(T_i - T_\infty)$$

$$\longrightarrow$$

$$\ln(T - T_\infty) = -kt + \ln(T_i - T_\infty)$$

$$\ln\left(\frac{T - T_\infty}{T_i - T_\infty}\right) = -kt$$

$$\frac{T - T_\infty}{T_i - T_\infty} = e^{-kt}$$

Condition 1: Temperature fall $32.5^\circ\ C \rightarrow 30.3^\circ\ C$

Setting the values, $T = 30.3, T_i = 32.5, T_\infty = 20$ & $t = 1\ hr$

$$\frac{30.3 - 20}{32.5 - 20} = e^{-k(1)}$$

$$e^{-k} = \frac{103}{125} \iff k = \ln\left(\frac{125}{103}\right)$$

Condition 2: Temperature fall $37^\circ\ C \rightarrow 32.5^\circ\ C$

Setting the values, $T = 32.5, T_i = 37, T_\infty = 20$, we get

$$\frac{32.5 - 20}{37 - 20} = e^{-kt}$$

$$e^{-kt} = \frac{25}{34} \iff t = \frac{1}{k}\ln\left(\frac{34}{25}\right)$$

Now, setting the value of k, we get

$$t = \frac{\ln\left(\frac{34}{25}\right)}{\ln\left(\frac{125}{103}\right)} = 1.5884\ hrs$$

Since, the murder took place at body temperature $37^\circ\ C$ & becomes $32.5^\circ\ C$ at 1 : 30 PM after $1.5884\ hrs = 95.3023\ minutes$

Hence, we conclude that

$$\boxed{\text{murder took place around } 11 : 54 : 42\ PM}$$

CONCLUSION

- Overall, we succeeded in calculating the time of death , the data was taken and fitted to the equation and it followed newton's law of cooling fairly well. Hence, we finally calculated the time of death.

- Hence, by the means of calculus, Newton's law of cooling has become a very efficient and accurate way to find out the time of death of a dead person. We can Practically observe that how fast the Body is cooling by the means of Calculus. This law is reasonably accurate approximation in almost each and every situation.

- Hence, we conclude our Study on Use of Calculus in finding the time of Death.

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