Assignment 2

1) $\int_{\cancel{1/10}\ 1/50}^{2} \frac{e^{2/w}}{w^2}\, dw$

$\int_{1/50}^{2} \frac{e^{2/w}}{w^2}\, dw$

Let $t = \frac{1}{w}$ $\qquad w \to 2$, $\; t \to 1/2$

$\frac{dt}{dw} = -\frac{1}{w^2}$ $\qquad w \to 1/50$, $\; t \to 50$

$dt = -\frac{dw}{w^2}$

$-dt = \frac{dw}{w^2}$

$\cancel{\int_{1/50}^{2} e^{2t} -dt}$ $\qquad = \int_{50}^{1/2} e^{2t} -dt$

$\cancel{-\int_{1/50}^{2} e^{2t} dt}$ $\qquad = -\int_{50}^{1/2} e^{2t} dt$

$\cancel{-\frac{1}{2}\left[e^{2t}\right]_{1/50}^{2}}$ $\qquad = -\frac{1}{2}\left[e^{2t}\right]_{50}^{1/2}$

$\cancel{-\frac{1}{2}\left[e^{2(2)} - e^{2(1/50)}\right]}$ $\qquad = -\frac{1}{2}\left[e^{2(1/2)} - e^{2(50)}\right]$

$\cancel{-\frac{1}{2}\left[e^{4} - e^{1/25}\right]}$

$\cancel{= -26.77866963.}$ $\qquad = -1/2\left(e^{1} - e^{100}\right)$

$= 1/2\left(e^{100} - e^{1}\right)$

$= 1.344058571 \times 10^{43}$

2) $\int \frac{2t^3+1}{(t^4+2t)^3}\, dt.$

Let

$w = t^4+2t$

$\frac{dw}{dt} = 4t^3+2$

$\frac{1}{2}\int \frac{4t^3+2}{(t^4+2t)^3}\, dt.$

$\frac{1}{2}\int \frac{1}{w^3}\, dw$

$\frac{1}{2}\int w^{-3}\, dw$

$\frac{1}{2} \frac{w^{-3+1}}{-3+1}$

$\frac{1}{2} \frac{w^{-2}}{-2}$

$-\frac{1}{4w^2}$

$\frac{-1}{4(t^4+2t)^2} + c$

3) $f'(x) = x^4+3x-9$

$f(x) = \int f'(x)$

$= \int x^4+3x-9$

$= \frac{x^5}{5} + \frac{3x^2}{2} - 9x + C$

4) $f(x) = \begin{cases} 6 & \text{if } x > 1 \\ 3x^2 & \text{if } x \leq 1 \end{cases}$

Evaluate $\int_{-2}^{3} f(x)\, dx$

$\int_{-2}^{3} f(x) = \int_{-2}^{1} 3x^2 dx + \int_{1}^{3} 6\ dx.$

$= \left[\frac{3x^3}{3}\right]_{-2}^{1} + (6x)_1^3$

$= (x^3)_{-2}^{1} + (6x)_1^3$

$= [1-(-2)^3] + [6(3)-6]$

$= [1-(-8)] + [18-6]$

$= (1+8) + (12)$

$= 9 + (12)$

$= \cancel{108}\ 21$

5) $\int 3(8y-1)e^{4y^2-y}\, dy.$

Let $4y^2-y$ be w

$\frac{dw}{dy} = 8y-1 \qquad dw = (8y-1)dy.$

$\int 3(8y-1)\, e^{4y^2-y}\, dy.$

$\int 3e^w\, dw$

$3e^w + c$

$3e^{4y^2-y} + c$

6 $f''(x) = 15\sqrt{x} + 5x^3 + 6$. $f(1) = -5/4$ $f(4) = 404$

$f(x) = ?$

$f'(x) = \int 15\sqrt{x} + 5x^3 + 6$

$= \frac{\cancel{15}^{5} x^{3/2}}{\cancel{3}/2} + \frac{5x^4}{4} + 6x + c.$

$= 10x^{3/2} + \frac{5x^4}{4} + 6x + c$

$f(x) = \int 10x^{3/2} + \frac{5x^4}{4} + 6x + c$

$f(x) = \frac{10x^{3/2+1}}{\frac{3}{2}+1} + \frac{\cancel{5}x^5}{\cancel{2}4\times\cancel{5}} + \frac{{}^{3}\cancel{6}x^2}{\cancel{2}} + cx + a$

$f(x) = \frac{{}^{2}\cancel{10}x^{5/2}}{\cancel{5}/2} + \frac{x^5}{4} + 3x^2 + cx + a$

$f(x) = 4x^{5/2} + \frac{x^5}{4} + 3x^2 + cx + a$

$f(1) = 4(1)^{5/2} + \frac{1}{4} + 3(1) + c(1) + a$

$-\frac{5}{4} = 4 + \frac{1}{4} + 3 + c + a$

$-\frac{\cancel{6}^{3}}{\cancel{4}2} - 7 = c + a$

$\frac{-3-14}{2} = c + a$

$-17/2 = c + a$

$a = -17/2 - c$

$f(4) = 4(4)^{5/2} + \frac{4^5}{4} + 3(4)^2 + 4c + a.$

$404 = 4 \times 4^2 \times \sqrt{4} + 4^4 + 3(4^2) + 4c - \frac{17}{2} - c$

$$404 = 128 + 256 + 48 + 3c - \frac{17}{2}$$

$$-28 + \frac{17}{2} = 3c$$

$$\frac{-56 + 17}{2} = 3c$$

$$\underline{\underline{c = -13/2}}$$

$$a = -\frac{17}{2} - c$$

$$a = -\frac{17}{2} + \frac{13}{2}$$

$$= -4/2$$

$$\underline{\underline{a = -2}}$$

$$\boxed{f(x) = 4x^{5/2} + \frac{x^5}{4} + 3x^2 - \frac{13}{2}x - 2}$$

→ partial differential equation

Formulate PDE from $f(x + iy) + g(x - iy)$

8 $\frac{dr}{d\theta} = \frac{r^2}{\theta}$ $\quad r(1)=2.$

$\frac{dr}{d\theta} = \frac{r^2}{\theta}$

$\frac{\theta}{d\theta} = \frac{r^2}{dr} \quad \frac{dr}{r^2} = \frac{d\theta}{\theta}$

Integrating both sides.

$\int \frac{dr}{r^2} = \int \frac{d\theta}{\theta}$

$\frac{-1}{r} = \log\theta + c$

$\frac{-1}{\log\theta + c} = r$

$r(1)=2.$

$\frac{-1}{r} = \log\theta + c$

$-1/r = \log 1 + c$

$-1/2 = 0 + c$

$\boxed{c = -1/2}$

$\boxed{\frac{-1}{r} = \log\theta - \frac{1}{2}}$ - Required solution

9 $\frac{dv}{dt} = 9.8 - 0.196v$

$\cancel{\frac{dv}{9.8 - 0.196v}} = \cancel{dt}. \quad \frac{dv}{dt} + 0.196v = 9.8$

$p = 0.196.$

$I.F = e^{\int Pdx} = e^{\int 0.196 v\, dv} = e^{0.196v}$

Solution

$v \times e^{0.196v} = \int 9.8 \times e^{0.196v}$

$ve^{0.196v} = 9.8e^{0.196v}$

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10 Find solution to following Initial value Problem (IVP)

$ty^t + 2y = t^2 - t + 1 \quad y(1) = 1/2.$

$t\frac{dy}{dt} + 2y = t^2 - t + 1$

$t\frac{dy}{dt} = t^2 - t + 1 - 2y$

$x = t^2 - t + 1 - 2y$

$\frac{dx}{dt} = 2t - 1 - 2\frac{dy}{dt}$

$\frac{2dy}{dt} = 2t - 1 - \frac{dx}{dt}$

$\frac{dy}{dt} = t - \frac{1}{2} - \frac{1}{2}\frac{dx}{dt}$

$\frac{dy}{dt} = t - \frac{1}{2} - \frac{1}{2}\frac{dx}{dt}$

$y = 1/2$ at $t = 1$

$t\frac{dy}{dt} + 2y = t^2 - t + 1$

$t\left(t - \frac{1}{2} - \frac{1}{2}\frac{dx}{dt}\right) = x$

$t^2 - \frac{t}{2} - \frac{t}{2}\frac{dx}{dt} = x$

$t^2 dt - \frac{t}{2}dt - \frac{t}{2}dx = x dt$

$\frac{t^3}{3} - \frac{t^2}{4} - \frac{tx}{2} = xt + c$

$t^3/3 - t^2/4 = \frac{3xt}{2} + c$

$\frac{t^3}{3} - \frac{t^2}{4} = \frac{3t}{2}\left(t^2 - t + 1 - 2y\right) + c$

$\frac{t^3}{3} - \frac{t^2}{4} = \frac{3}{2}t(t^2 - t + 1 - 2y) + c$

$c = \frac{t^3}{3} - \frac{t^2}{4} - \frac{3t^3}{2} + \frac{3t^2}{2} - \frac{3t}{2} + 3yt$

$c = -\frac{7t^3}{96} + \frac{10t^2}{8} - \frac{3t}{2} + 3yt$

$c = -\frac{7}{6} + \frac{10}{8} - \frac{3}{2} + \frac{3}{2}$

$c = 2/24$

$c = 1/12$

12 Find Taylor Series for $f(x) = x^3 - 10x^2 + 6$ for $x=3$

$x=3 \quad f(3) = -57$

$f'(x) = 3x^2 - 20x$ $\qquad f'(3) = -33$

$f''(x) = 6x - 20$ $\qquad f''(3) = -2$

$f'''(x) = 6$ $\qquad f'''(3) = 6$

$f''''(x) = 0$ $\qquad f''''(3) = 0.$

$$f(x) = F(3) + (x-3)' f'(3) + \frac{(x-3)^2}{2} f''(3) + \frac{(x-3)^3}{6} f'''(3)$$

$$= -57 + (x-3)(-33) + \frac{(x-3)^2}{2}(-2) + \frac{(x-3)^3}{6} 6$$

$$\boxed{f(x) = -57 - 33(x-3) - (x-3)^2 + (x-3)^3}$$

13 Find the Maclaurin series for $(1+x)^u$

$f(x) = (1+x)^u$

$f'(x) = u(1+x)^{u-1}$ $\qquad f'(0) = u(1+0)^{u-1}$

$= u(1)^{u-1} = u.$

$f''(x) = u \times \frac{d}{dx}(1+x)^{u-1} + (1+x)^{u-1}\frac{d}{dx}u$ $\qquad f''(0) = u(u-1)(1+0)^{u-1-1}$

$= u^2 - u(1)$

$= u^2 - u$

$f'''(0) = u(u-1)(u-2)(1+0)^{u-3}$

$= u(u-1)(u-2)1$

$= u(u-1)(u-2)$

Maclaurin series

$$f(0) + f'(0)x + f''(0)\frac{x^2}{2!} + f'''(0)\frac{x^3}{3!}$$

$$1 + ux + \frac{u^2-u}{2}x^2 + \frac{u(u-1)(u-2)}{6} \ldots$$

14 Determine the Maclaurin series for $f(x)=\sqrt{1+x}$

$f(x)=\sqrt{1+x}$

$f(0)=\sqrt{1+0}=\sqrt{1}$

$f(x)=(1+x)^{1/2}$

$f'(x)=\frac{1}{2}(1+x)^{1/2-1}$

$f'(0)=\frac{1}{2}(1+0)^{-1/2}=\frac{1}{2}(1)^{-1/2}=\frac{1}{2}$

$=\frac{1}{2}(1+x)^{-1/2}$

$f''(0)=-\frac{1}{4}(1+0)^{-3/2}=-\frac{1}{4}$

$f''(x)=-\frac{1}{4}(1+x)^{-3/2}$

$f'''(0)=\frac{3}{8}(1+0)^{-5/2}=\frac{3}{8}.$

$f'''(x)=-\frac{1}{4}\left(-\frac{3}{2}\right)(1+x)^{-5/2}$

$=\frac{3}{8}(1+x)^{-5/2}$

$f(x)=f(0)+\frac{x^1}{1!}f'(0)+\frac{x^2}{2!}f''(0)+\frac{x^3}{3!}f'''(0)$

$f(x)=\sqrt{1}+x\left(\frac{1}{2}\right)+\frac{x^2}{2}\left(-\frac{1}{4}\right)+\frac{x^3}{\cancel{6}\,2}\left(\frac{\cancel{3}\,1}{8}\right)$

$=\sqrt{1}+\frac{x}{2}-\frac{1}{8}x^2+\frac{x^3}{16}$

15 Find the Taylor series for $f(x)=e^{-6x}$ about $x=-4$.

$f(x)=e^{-6x}$

$f(-4)=e^{-6(-4)}=e^{24}$

$f'(x)=e^{-6x}(-6)$

$f'(-4)=e^{-6(-4)}(-6)=-6e^{24}$

$f''(x)=e^{-6x}(36)$

$f''(-4)=e^{24}(36)$

$f'''(x)=e^{-6x}(-216)$

$f'''(-4)=-216e^{24}$

$f(x)=f(-4)+\frac{x+4}{1}f'(-4)+\frac{(x+4)^2}{2!}f''(-4)+\frac{(x+4)^3}{3!}f'''(-4).$

$f(x)=e^{24}+(x+4)-6e^{24}+\frac{(x+4)^2}{2}36e^{24}+\frac{(x+4)^3}{6}-216e^{24}$

$=e^{24}-6e^{24}(x+4)+\cancel{36}\,18\,e^{24}(x+4)^2-\cancel{216}\,36\,e^{4}(x+4)^3$

Q16 Find the Taylor series for $f(x) = \ln(3+4x)$ about $x=0$.

$f(x) = \ln(3+4x)$ $\qquad$ $f(0) = \ln 3$

$f'(x) = \dfrac{1(4)}{3+4x}$ $\qquad$ $f'(x) = \dfrac{4}{3}$

$f''(x) = \dfrac{-(4)(4)}{(3+4x)^2}$ $\qquad$ $f''(x) = \dfrac{-16}{(3)^2} = \dfrac{-16}{9}$

~~$f'''(x)$~~

$f''(x) = -16(3+4x)^{-2}$ $\qquad$ $f'''(x) = \dfrac{32}{3(3)^3}$

$f'''(x) = \dfrac{+32(3+4x)^{-3}}{3}$

$\qquad = \dfrac{32}{81}$

$f'''(x) = \dfrac{32}{3(3+4x)^3}$

$$f(x) = f(0) + \frac{x}{1}f'(0) + \frac{x^2}{2!}f''(0) + \frac{x^3}{3!}f'''(0)$$

$$= \ln 3 + x\left(\frac{4}{3}\right) + \frac{x^2}{\cancel{2}}\left(\frac{-\cancel{16}^{\,8}}{9}\right) + \frac{x^3}{\cancel{6}_{\,3}}\left(\frac{\cancel{32}^{\,16}}{81}\right)$$

$$= \ln 3 + \frac{4x}{3} + \frac{-8x^2}{9} + \frac{16x^3}{243}$$