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PRLI Assignment -1 Roll No. 703

Q1
i) $(Ia)_{\overline{n}|} = v + 2v^2 + 3v^3 + \dots nv^n$ — (A)

$(1+i)(Ia)_{\overline{n}|} = 1 + 2v + 3v^2 + \dots + nv^{n-1}$ — (B)

(B) — (A)

$= i(Ia)_{\overline{n}|} = 1 + v + v^2 + v^3 + \dots v^{n-1} - nv^n$

$= i(Ia)_{\overline{n}|} = \ddot{a}_{\overline{n}|} - nv^n$

$= Ia_{\overline{n}|} = \dfrac{\ddot{a}_{\overline{n}|} - nv^n}{i}$

ii) cost of education = 30 00000
inflation = 6%.pa

Contribution = 200 000 pa
interest = 10%.
PV of contribution.

$200000\, \ddot{a}_{\overline{12}|} + X(Ia)_{\overline{9}|}$ } @ i=10%

$= 200000 \times 6.759026 + x \times 25.18046$

$= 1351805.20 + 25.18046x$

P.V of higher education : $3000000 \times 1.06^{10} \times v^{10}$ @ 10%

$= 2071360$

$x = 28575$

Q2.

$$\begin{tikzcd}
\boxed{n} \arrow[rr, "\sigma_x"] \arrow[dr, "\mu_x"'] & & \boxed{i} \arrow[dl, "\nu_x"] \\
& \boxed{d} &
\end{tikzcd}$$

$i = 5\%$.

$\mu_x = 0.004$

$\nu_x = 0.005$

$\sigma_x = 0.002$

$x = 40$

$n = 20$

$S = 100000$ (immediately)

$$EPV = 100000 \int_0^{20} v^t \cdot {}_tp_{40}^{n} \left(\sigma_{40+t} + \mu_{40+t}\right) dt$$

$$= 100000 \int_0^{20} e^{-\ln(1.05)t} \cdot e^{-0.006t} \cdot 0.006\, dt$$

$$= 600 \int_0^{20} e^{-\ln(1.05)t - 0.006t} \cdot dt$$

$$= \frac{600}{0.5497} \left[e^{-0.5497t}\right]_0^{20}$$

$EPV = 7290.3$

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Q3. $x = 40$

$n = 20$

$i = 4\%$

AM92 select - disability

AM92 ult = illness

i.)

EPV of prem = $P\ddot{a}^{(12)}_{40:\overline{20|}}$

$= P\left[\ddot{a}_{40:\overline{20|}} - \frac{11}{24}\left(1 - \frac{D_{60}}{D_{40}}\right)\right]$

$= P\left[13.927 - \frac{11}{24} \times \left(1 - \frac{882.85}{2052.96}\right)\right]$

$= 13.66576724\, P$ — ①

EPV of claims :-

$750000\, A^{1}_{40:\overline{20|}} + 50000\, A^{1}_{[40]:\overline{20|}}$

$\downarrow$ $A_{40:\overline{20|}} - \frac{D_{60}}{D_{40}}$

$\downarrow$ $A_{[40]:\overline{20|}} - \frac{D_{60}}{D_{[40]}}$

$= 427714.965824$ — ②

EPV of expenses

$0.03 \times 427714.965824 + 0.35P\,\ddot{a}^{(12)}_{40:\overline{1|}}$

$+ 0.05P\left[\ddot{a}^{(12)}_{40:\overline{20|}} - 1\right]$

$+ 0.1 P \times \ddot{a}_{40:\overline{20}|} \times 1.04^{0.5} + 500$ — (3)

3)
~~Soln~~

(1) = (2) + (3)

Solving for $P \Rightarrow 39009.7$

$\therefore$ Monthly $P = \underline{\underline{3250.8}}$

ii. Policies with only accidental claims

$7500000 \, A^{1}_{45:\overline{5}|} - 39009.7 \, \ddot{a}^{(12)}_{45:\overline{5}|}$

$= 269387.1975 - 434750.6352$

${}_5V_1 = 165363.4377$

Policies with only illness claims

$5000000 \, A^{1}_{45:\overline{15}|} - 390097 \, \ddot{a}^{(12)}_{45:\overline{15}|}$

$= 5000000 \times 0.055918293 - 434750.6352$

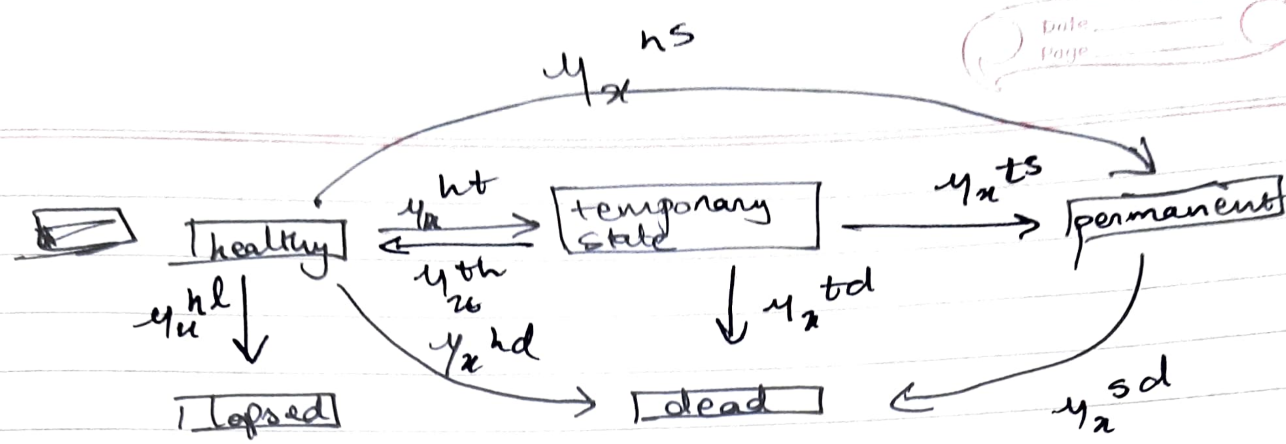
${}_5V_2 = 255159.1302$

Policies with no claims

$= 7500000 + 5000000 \times A^{1}_{45:\overline{15}|} - 390097 \, \ddot{a}_{45:}$

${}_5V_3 = 14228.0273$

Q4. i)



ii) $n = 15$

$x = 50$

$$EPV\,(prem) = EPV\,(benefit)$$

$$EPV\ benefit = 10000 \int_0^{15} v^u \cdot {}_u p_{50}^{ht} \, du$$

$$+ 20000 \int_0^{15} v^u \, {}_u p_{50}^{hs} \, du$$

$$+ 100000 \int_0^{15} v^u \left({}_u p_{50}^{hh} \cdot \mu_{50+u}^{hd} + {}_u p_{50}^{ht} \cdot \mu_{50+u}^{td} + {}_u p_{50}^{hs} \cdot \mu_{50+u}^{sd} \right) du$$

$$EPV\ of\ premium = P \times \int_0^{15} v^u \cdot {}_u p_{50}^{hh} \, du$$

$$\therefore P = \frac{\left[100000 \int_0^{15} v^u \left[{}_u p_{50}^{hh} \cdot \mu_{50+u}^{hd} + {}_u p_{50}^{ht} \cdot \mu_{50+u}^{td} + {}_u p_{50}^{hs} \, \mu_{50+u}^{sd} \right] du + 10000 \int_0^{15} v^u \, {}_u p_{50}^{ht} \, du + 20000 \int_0^{15} v^u \, {}_u p_{50}^{hs} \, du \right]}{\int_0^{15} v^u \, {}_u p_{50}^{hh} \, du}$$

Q5

i) Gross premium for the Policy:

EPV premiums $= P\ddot{a}_{[40]:25} = 15.887P$

EPV expenses $= 1200 + 500 \cancel{\times} \bar{A}_{[40]} + 2\% P \times (\ddot{a}_{[40]:25} - 1)$

$= 1200 + 500 \times (1.04)^{0.5} \times 0.23041$
$+ 0.02 \times P \times (15.887 - 1)$

$= 1317.487 + 0.2977 P$

EPV of Benefits $= 200000 \left(v^{0.5} q_{[40]} \right.$
$+ v^{1.5}(1.04) {}_{1|}q_{[40]}$
$\cancel{+ v^{2.5}(1.04)^2 {}_{2|}q}$
$\left. + v^{2.5}(1.04)^2 {}_{2|}q_{[40]} + \cdots \right)$

$= \dfrac{200000}{1.04^{0.5}} = 196116.14$

Using equivalence principle,

$15.88P = 196116.14 + \cancel{P}1317.487 + 0.29774P$

$P = 12664$

ii. Revised bonus rates :-

EPV of benefits $= 2,00,000 \times x\% (\bar{IA})_{[40]} + 200000 \times (1-x\%) \bar{A}_{[40]}$

$= (1.04)^{0.5} \left[200000 \times x\% (IA)_{[40]} + 200000 \times (1-x\%) A_{[40]} \right]$

EPV of benefits. $=$ ~~₹~~ 196116.144

$\therefore$ Equating both, $x\% = 9\%$.

iii. $P\ddot{a}_{40:25} = 200000 \bar{A}_{40}$

$P = 2960.54$

Net premium reserve at time 10

${}_{10}V = 300000 \bar{A}_{50} - P\ddot{a}_{50:15}$

$= 300\,000 \times 1.04^{0.5} \times 0.32907 - 2960.54 \times 11.25$

$= 67361.10$

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Q6. Let force of decrement be μ_x, force of decrement be σ_x

$$V = 100000 \int_0^{20} v^t \; {}_tp_{45}^{hh} (\mu_{45+t} + \sigma_{45+t}) \, dt$$

$$100000 \int_0^{20} e^{-\ln(1.05)t} \cdot {}_tp_{45+t}^{hh} \cdot 0.008 \, dt$$

$${}_tp^{hh} = \exp\left(-\int^{46} {}_{45+t} (0.008 \, ds)\right)$$

$$= e^{-0.008t}$$

$$V = 800 \times \int_0^{20} e^{-\ln(1.05)t} \cdot e^{-0.008t} \, dt$$

$$= 800 \int_0^{20} e^{-0.05679t} \, dt$$

$$= 800 \left[\frac{-e^{-0.05679t}}{0.05679}\right]_0^{20}$$

$$= 800 \, (-5.65531 + 17.60873) = 9562.8$$

Value of RoP

$$P \times e^{\int_{45}^{65} (0.006 + 0.002) \, dt} \cdot v^{20}$$

$$P \times e^{-0.16} \times 0.37689$$

Premium = Value of Benefits on Death & Cancer + Value of ROP + Commission + Expenses + Profit Margin

$P = 9562.8 + 0.321164 \times P + 0.02 \times P + 0.02 \times P$
$+ 0.1 \times P$

$P = 17747.15$

Q7. PV of net cashflows = PV of Premium − PV of benefits − PV of expenses − PV of commission

Let max initial commission be I

P.V of Premium $= 30000 \cdot \ddot{a}_{[45]:20}$

$= 30000 \times 11.888$

$= 356640$

PV of benefits $= 1000000 \times A_{[45]:20}$

$= 327110$

PV of expenses $= 5000 + 2.5\%$ of $30000 \times (\ddot{a}_{[45]:20} - 1)$

$+ 500 \times p_{[45]} \times 1.06^{-1} \times (1 + 1.019231 \times v \times p_{[46]} + 1.019231^2 \times v^2 \times {}_2p_{[46]}$

$\ldots + 1.019231^{18} \times v^{18} \times {}_{18}p_{[46]})$

$= 5000 + 2.5\%\ 30000\ (\ddot{a}_{[45]:20} - 1)$

$+ 500 \times p_{[45]} \times 1.06^{-1} \times \ddot{a}_{[46]:19}$ @4%

$= 19438.$

P.V of renewal commissions

$= 2\%$ of $30000 \times (\ddot{a}_{[45]:20} - 1)$

$\times 11.888 - 1$

$=$

$= 6533$

PV of net CF $=$ ~~3557 - I~~ $356640 - 327110 - 19438 - 6533$

$= 3559 - I$

Margin $= \dfrac{\text{PV of net CF}}{\text{Annual Premium}}$

$= \dfrac{3559 - I}{30000} = 10\%.$

$I =$ Rs. 559

Q8. 1) The assumption of indepence of decrements is used to derive single decrement table from a multiple decrement table.

Mathematically:

$(a\mu)_x^j = \mu_x^j$ for all j and all x

When we look at transition intensities (forces of decrement) we are looking at infiniteomilly small time interval for which there is only time for one decrement. Thus it is reasonable to assume that the independent and dependent forces of decrement are equal.

ii. In case of term assurance, it is likely that people who lapse their policies have a lower than average mortality.

iii. a) $1000 \int_0^5 e^{-\delta t} \left({}_tp_{45}^{\,PI} \mu_{45+t} + {}_tp_{45}^{\,PP} \nu_{45+t} \right) dt$

b) $250 \int_0^5 e^{-\delta t} \; {}_tp_{45}^{\,IP} \nu_{45+t} \, dt$

c) $150 \int_0^5 e^{-\delta t} \; {}_tp_{45}^{\,II} \, dt$

d) $250 \int_0^5 e^{-\delta t} \; {}_tp_{45}^{\,PP} \nu_{45+t} \, dt$

iv. $(aq)_{45}^{c} = \frac{0.05}{0.18}\left(1 - e^{-0.18}\right) = 0.045758$

$(aq)_{45}^{s} = \frac{0.13}{0.18}\left(1 - e^{-0.18}\right) = 0.118972$

$(ap)_{45} = 1 - 0.045758 - 0.118972 = 0.835270$

$(aq)_{46}^{c} = \frac{0.06}{0.16}\left(1 - e^{-0.16}\right) = 0.055446$

$(aq)_{46}^{s} = \frac{0.1}{0.16}\left(1 - e^{-0.16}\right) = 0.092410$

$(ap)_{46} = 0.8521114$

Possibility of being inforce at age 47 = $0.835270 \times 0.8521114$
$= 0.711770$

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Q9.

i. Under with profits policies the contract between policyholder and the company is such that, whatever is the surplus arising on the underlying fund, would be shared between policyholders and shareholders in a predetermined manner.

Now, as the surplus arises over future years the same is shared with policyholders in form of bonuses and hence the need of bonuses under with profit policies.

ii. Investment surplus

Expense surplus

Mortality surplus

Lapse/surrender surplus

Reinsurance surplus

iii. Terminal bonus would be a strategy to build lower guarantees over the lifetime of with profit policies.

If the proportion of RB and TB is skewed towards TB then it would mean lower guarantees to policyholders and hence companies would have greater freedom in terms of choosing investment assets.

By doing this, they can take risk of investing in real assets like equities and real estate which can lead to better final results to policyholders, which would be ~~re~~distributed through higher TB.

Q10. Let monthly premium be P.

EPV of premium $= 12 \times P \times \ddot{a}^{(12)}_{50:\overline{10|}}$ at 6%.

$$\ddot{a}^{(12)}_{50:\overline{10|}} = \ddot{a}_{50:\overline{10|}} - \frac{11}{24} \times (1 - {}_{10}p_{50} \times v^{10})$$

$\uparrow$
7.694

$$\ddot{a}^{(12)}_{50:\overline{10|}} = 7.694 - \frac{11}{24} \times \left(1 - 1.06^{-10} \times \frac{9287.2164}{9712.0728}\right)$$

$= 7.4804$

EPV of premiums $= 89.7648 \times P$

EPV of benefits $= 10{,}00{,}000 \times \big(q_{50} \times v + 1.019231 \times v^2 \times {}_{1|}q_{50}$

$+ \ldots + 1.019231^{14} \times v^{15} \times {}_{14|}q_{50}$

$+ 1.019231^{15} \times v^{15} \times {}_{15}p_{50}\big)$

$= \frac{1000000}{1.019231} \times \big(q_{50} \times v \times 1.019231 + 1.019231^2 \times v^2 \times {}_{1|}q_{50} + \ldots$

$\ldots + 1.01931^{15}\, v^{15}\; {}_{14|}q_{50}$

$+ 1.01931^{15} \times v^{15} \times {}_{15}p_{50}\big)$

$\longrightarrow$

Converting to 4%.

$$\text{EPV of benefits} = 1000000 \times \left(\frac{1}{1.019231}\right) \times A_{50:\overline{15}|} \ @ 4\%$$

$$+ 1000000 \times v^{15} \text{ at } 6\% \times {}_{15}p_{50}$$

$$\times (1.019231^{15} - 1.019231^{14})$$

$$\text{EPV} = 566004.1$$

EPV of expense & commission.

$$= \frac{20+35}{100} \times 12 \times P + \left(\frac{2+15}{100}\right) \times {}_1p_{50} \times 1.06^{-1}$$

$$\times 12 P \times \ddot{a}^{(12)}_{51:\overline{9}|}$$

$$+ 2500 + 400 \times {}_1p_{50} \times 1.06^{-1} \times \ddot{a}_{51:\overline{14}|} \text{ at } 6\%.$$

$$= 9.3333P + 6115.06$$

$$\therefore \text{EPV of premiums} = \text{EPV of benefits} + \text{EPV of expense \& commission}$$

$$89.7648 \times P = 566004.1 + 9.333P + 6115.06$$

$$80.4315 \times P = 572119.16$$

$$\underline{\underline{P = 7113.12}}$$

ii. Sum Assured at end of 10th year + Accrued bonuses

$X = 1000000 \times 1.02^{10}$

$X = 1218994$

PV of future benefits at end of 10th year

$= \frac{X}{1.05} \times q_{60} + \frac{X}{1.05^2} \times 1.04 \times {}_{1|}q_{60} + \frac{X}{1.05^3} \times 1.04^2 \times {}_{2|}q_{60}$

$+ \frac{X}{1.05^4} \times 1.04^3 \times {}_{3|}q_{60} + \frac{X}{1.05^5} \times 1.04^4 \times {}_{4|}q_{60}$

$+ \frac{X}{1.05^5} \times 1.04^5 \times {}_{5}p_{60}$

$= 0.952948 \times X$

$= 11.60774$

PV of expenses $= 400 \times \ddot{a}_{60:\overline{5}|}$ @5%.

$= 400 \times \left(1 + 1.05^{-1} \times \frac{l_{61}}{l_{60}} + 1.05^{-2} \frac{l_{62}}{l_{60}} + 1.05^{-3} \frac{l_{63}}{l_{60}} + 1.05^{-4} \frac{l_{64}}{l_{60}}\right)$

$= 1787.39$

Reserves required = PV of benefits + PV of expenses

$= \underline{1162561}$

11.

$i = 6\%$

$SA = 400000$

Simple Bonus = 40 per 1000 SA

Net premium ~~...~~ prospective reserve at 7

EPV of premium = $P\ddot{a}_{30:\overline{20}|}$

EPV of benefit = ~~$P\ddot{a}_{30}$~~ $400000\, A_{30:\overline{20}|}$

$\therefore P\ddot{a}_{30:\overline{20}|} = 400000\, A_{30:\overline{20}|}$

$P = \dfrac{400000\, \cancel{A_{30}}\,(0.315817)}{12.08705}$

$P = 10451.42$

Total bonus at time 7 $= 7 \times 0.04 \times 400000$
$= 112000$

SA at time 7 $= 400000 + 112000 = 512000$

Reserve at time 7

$= 512000\, A_{37:\overline{13}|} - 10451.42\, \ddot{a}_{57:\overline{13}|}$

$= 512000\,(0.47638) - 10451.42\, \cancel{\ddot{a}}\, 9.335835$

$= 143926.89$