Date
Page

Assignment

Ans 10)

$$\int_1^\infty \int_1^\infty R x^{-\alpha} e^{-y/\beta} \, dx \, dy = 1$$

$$\int_1^\infty R x^{-\alpha} e^{-y/\beta} \, dx$$

$$= \cancel{\int_1^\infty} \left[\cancel{Rx} \quad R e^{-y/\beta} \left[\frac{x^{-\alpha+1}}{-\alpha+1} \right]_1^\infty \right.$$

$$= \int_1^\infty R e^{-y/\beta} \cdot \frac{1}{1-\alpha} \, dy = 1$$

$$= \frac{R}{1-\alpha} \int_1^\infty \frac{e^{-y/\beta}}{1} dy = 1$$

$$= \frac{R}{1-\alpha} \left[\beta e^{-y/\beta} \right]_1^\infty = 1$$

$$= \beta e^{-1/\beta} \cdot \frac{R}{1-\alpha} = 1$$

$$= \left(\frac{\beta e^{-1/\beta}}{1-\alpha} \right) \cdot R = 1$$

$$R = \frac{1-\alpha}{\beta e^{-1/\beta}}$$

Ans 9)

X	Y	
mean = 3	~~mean = 4~~	
S.d = 2	S.d = 1	corr coeff = − 0.3
Variance = 4	Variance = 1	

$Z = X + Y$

$E(Z) = 1\,E(X) + 1\,(E(Y)$

$= 3 + 4 = 7$

Covariance =

$\text{cov}(X, Z) \;/= E(XZ) \mp E(X)E(Z)$

$$\text{Correlatio} = \frac{Cov(X, Y)}{\sqrt{var(X) \cdot var(Y)}}$$

$$-0.3 = \frac{Cov(X, Y)}{\sqrt{4 \times 1}}$$

$$-0.3 \times 2 = Cov(X, Y)$$

$$\boxed{-0.6 = Cov(X, Y)}$$

$$-0.6 = E(XY) - E(X) E(Y)$$

$$-0.6 = E(XY) - (4)(3)$$

$$12 - 0.6 = E(XY)$$

$$11.4 = E(XY)$$

i) $Cov(X, Z) = Cov(X, X+Y)$

$$= Cov(X, X) + Cov(X, Y)$$

$$= 4 - 0.6 \qquad [Cov(X, X) = Var$$

$$= 3.4$$

ii) $Var(Z) = Var(X+Y) = Var(X) + var(Y) + 2\,cov(X, Y)$

$$= 4 + 1 + 2(-0.6)$$

$$= 5 - 1.2$$

$$= 3.8$$

Ans 8) i) $E(Y) = E(E(\cancel{Y|X}))$

$$\cancel{E(Y|X=x) = 3x+1}$$

$$\cancel{= E(3x+1)}$$

$$=$$

$$= E(E(Y|X)$$

$$= E(3x+1)$$

$$= \frac{3}{2}(3x+1)$$

$$= 9/2\, x + 3/2$$

ii) $V(Y) = E(V(Y|X)) + V(E(X|Y)$

$$= E(8x^2+5) + V(3x+1).$$

Ans 7) i) $var(X| y=2) = E(x^2 | y=2) - [E(X|y=2)]^2$

$$= (0.3)^2(2) + (3)(0.1)^2 + 4(0.2)^2 - (0.6 + 0.3 +$$

$$= \cancel{0.37}$$

$$= .2.52$$

ii) $\int_{u/2}^{2}\int_{0}^{1} \frac{48}{67}(2uv - u^2)\,du\,dv$

$f_u(u) = \int_{u/2}^{2} \frac{48}{67} \int_{0}^{1} (2uv - u^2\,du\,dv$

$= \frac{48}{67}\left[vu^2 - \frac{u^3}{3}\right]_0^1$

$= \int_{u/2}^{2} \frac{48}{67}\left[v - \frac{1}{3}\right] dv$

$= \int_{u/2}^{2} \frac{48}{67} \frac{v^2}{2} - \frac{1}{3}v\,dv$

$= \frac{48}{67}\left[\frac{v^2}{2} - \frac{1}{3}v\right]_{u/2}^{2}$

$= \frac{48}{67}\left(\left[2 - \frac{2}{3}\right] - \left[\frac{u^2}{8} - \frac{u}{6}\right]\right)$

$= \frac{48}{67}\left(\frac{4}{3} - \left(\frac{6u^2 - 8u}{48}\right)\right)$

$= \frac{48}{67} \times \frac{64 - 6u^2 - 8u}{48}$

$= \frac{64 - 6u^2 - 8u}{67}.$

$E(U/V = v) = \int_0^1 u \cdot \left(\frac{64 - 6u^2 - 8u}{67}\right) du$

Ans 6) $X \sim Poi(\lambda)$ & $Y \sim Poi(\mu)$

~~$X - Y$~~ = Case 1.

MGF of $X = e^{\lambda(e^t - 1)}$

MGF of $Y = e^{\mu(e^t - 1)}$

$X - Y = e^{\lambda(e^t - 1)} - e^{\mu(e^t - 1)} \neq Poi(\lambda - \mu)$ which is not a poisson distribution

Case 2.

$X + Y \sim Pois(\lambda + \mu)$

MGF of $X = e^{\lambda(e^t - 1)}$

MGF of $Y = e^{\mu(e^t - 1)}$

$X + Y = e^{\lambda(e^t-1)} + e^{\mu(e^t-1)}$

$= e^{\lambda+\mu(e^t-1)} = Poi(\lambda + \mu)$

$\therefore$ Sum of two independen poisson distribution is a poisson distribution.

Ans 5) $f(x,y) = ax(x+y)$

$E(Y|X) = \sum_{x} x \cdot f_{x,y}(x,y)\, dx$

Ans 4) $P(V = 4)$

$G_V(t) = p^k (1 - qt)^{-k}$

$P(V = 4) = p^{k+4} (1 - qt)^{-k-4}$

Ans 3) i) $f(x) = \lambda e^{-\lambda(x - \alpha)}$

MGF $= \left(1 - \frac{t}{\lambda}\right)^{-\alpha}$ (which is MGF for gamma).

ii) mean $\Rightarrow \frac{\alpha}{\lambda}$

Variance $\Rightarrow \frac{\alpha}{\lambda^2}$.

Date
Page

Ans 2) i) $M_y(t) = M_N(\ln M_x(t))$

$= M_N\left(\log \left\{ \cancel{pe^t} \left(\frac{0.9e^t}{1-0.1e^t}\right)^3\right)\right)$

MGF of Gamma $= \left(1-\frac{t}{2}\right)^{-6}$

Ans 1) Home

$\mu = 800$

$\sigma^2 = 10000$

Car

Mean $= 1200$

$\sigma^2 = 90{,}000$

$Z = \frac{800-800}{10{,}000}$

$= 0$

$Z = \frac{800-1200}{90{,}000}$

$= \frac{400}{90{,}000} = 0.0044$

$P(Z > 0.004444)$