INSTITUTE OF ACTUARIAL & QUANTITATIVE STUDIES

**Subject:** Probability and Statistics

**Chapter:** Unit 1 & 2

**Category:** Assignment Questions

**1.** Define the following for a discrete random variable $X$.
i) The kth moment.
ii) The kth moment about $\alpha$.
iii) The kth central moment.
iv) The coefficient of skewness.

**2.** A newly established life insurance company is analyzing the experience of policy withdrawal by the policyholders of a portfolio of 10,000 policies, on basis of the channel through which the policy was sold. The table below shows the split of the number of policies sold by the respective channels and the probability of a policyholder withdrawing the policy in a year

| Channel | Agency | Bank | Online |
|---|---|---|---|
| Probability ($p$) | 0.05 | 0.08 | 0.14 |
| Number of policies ($n$) | 2000 | 3500 | 4500 |

It can be assumed that the withdrawal by any individual policyholder during any year is independent of withdrawal by other policyholders of same or different channel.

i) Calculate the probability that a randomly selected policyholder will withdraw in a particular year.

ii) Calculate the probability that a randomly selected policyholder will withdraw in a particular year given that the policy was not sold through online channel.

iii) Calculate the probability that a randomly selected policy was sold by Bank given that the policyholder withdrew last year.

**3.** The following are the marks scored by 24 students in a theory paper conducted for 60 marks:

| | | | |
|---|---|---|---|
| 22 | 31 | 26 | 22 |
| 19 | 21 | 33 | 27 |
| 53 | 27 | 34 | 27 |
| 46 | 30 | 21 | 30 |
| 17 | 60 | 36 | 32 |
| 26 | 33 | 27 | 33 |

i) Display the above data in a stem and leaf diagram.

ii) Calculate the median and mode.

iii) Calculate the interquartile range.

**4.** Obtain the recursive relation for the binomial distribution (n,p) of the form

P(X=x)=g(x, n, p) P(X=x-1) ; x=1,2, ….., n ; 0<p<1

where g(x,n,p) is a general function of x,n and p

**5.** Suppose that 30% of passengers in a queue at a taxi stand at Town A wish to go to Town B which is 150km far away.

i) Calculate the probability that a four- seater taxi owner (excluding driver) will need to ask 15 passengers to get 4 passengers for the trip to Town B.

ii) Find the average number of persons to be asked in order to fulfil the taxi needs.



**6.** A continuous random variable X has probability density function

$$f(x) = \frac{3x^2}{\theta^3}\ ; 0 < x < \theta \text{ and } 0\ otherwise.$$

i) Calculate the mode and median of X.

ii) Calculate the probability that $X$ is less than the ratio of the mode to the median of $X$.

**7.** The table below shows the maximum monthly temperatures (in ˚Celsius) recorded in a year for two cities, A and B.

| *Month* | *Jan* | *Feb* | *Mar* | *Apr* | *May* | *Jun* | *Jul* | *Aug* | *Sep* | *Oct* | *Nov* | *Dec* |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *City A* | 21 | 26 | 26 | 29 | 31 | 33 | 35 | 32 | 30 | 26 | 21 | 20 |
| *City B* | 18 | 25 | 25 | 32 | 37 | 68 | 42 | 32 | 29 | 25 | 20 | 16 |

i) Draw boxplot diagrams for the maximum monthly temperatures of these cities.

ii) Use the boxplot diagrams to compare and contrast the two data sets.

**8.** The random variable X has a lognormal distribution with the same mean and variance as that of the $\chi_9^2$ distribution.
Calculate $P(X > 9)$.

**9.** In a bakery, the time taken to prepare an exotic cake is normally distributed with mean 2 hours and standard deviation 15 minutes.

Calculate the probability that the time taken for two randomly selected exotic cakes differs by no more than 25 minutes.

**10.** Assume that a company has 99 employees and they are drawing the same salary.

i) Calculate the arithmetic mean and the standard deviation of salaries of these employees.

Later, one more employee has joined the company and his salary is Rs 1000 more than those of the existing employees.

ii) Calculate the arithmetic mean and the standard deviation of salaries of the 100 employees and comment.



**11.** The results of an experts committee on an investigation of car accidents, only due to tyre burst or collision with road divider, are summarized below.
P (Accidents due to tyre burst) = 0.6
P (Accidents due to collision with road divider) = 0.4

It is known from past records that the death casualty in car accidents due to tyre burst and on collision with road divider are 30% and 50% respectively.

i) Calculate P [Tyre burst/a death casualty due to car accident]

ii) Comment on the most probable cause of death, between the two, in car accidents.

**12.** A secretary is given 100 computer passwords and only one which is correct opens a file. Since the secretary has no information on the correct password, she tries to open using one of the passwords. She randomly chooses one and discards it if incorrect until she finds the correct one.

i) Calculate the probability that she obtains the correct password in the third attempt.

A security system has been set up so that if three incorrect passwords are tried before the correct one, the computer file is locked and access to it is denied.

ii) Calculate the probability that the secretary will gain access to the file.

The secretary selects a password tries it and if it does not work, puts it back with the other passwords before randomly selecting a new password.

iii) Calculate the probability that the correct password is found on the tenth attempt.

**13.** An academician proposes that the joint distribution of a number of weeks of study leave (X) and the proportion of questions answered correctly (Y) is given by:

$$f_{XY}(x,y) = \frac{9}{10}xy^2 + \frac{1}{5} \; for \; 0 \le x \le 2, 0 \le y \le 1$$

i) Determine the Marginal distributions of X and Y.

ii) A student is selected at random, what would be the expected number of weeks of study leave and expected proportion of questions answered correctly.

iii) Compute the covariance of X and Y

**14.** Let X and Y be iid random variables from an exponential distribution with mean 0.5.

i) Defining Z = Min (X ,Y), obtain the cumulative distribution function of Z.

ii) Hence, find the mean of Z.

**15.** Let the random variables X and Y have joint probability density function (pdf)

$$f_{X,Y}(x,y) = \frac{12}{5}(x^2y + xy)\,; 0 < x,y < 1.$$

i) Find the marginal pdfs of X and Y

ii) Check for the independence of X and Y

iii) Compute E(X) and E(Y)

iv) Compute E(X/Y) and Var(X/Y). Hence, verify E (E(X/Y)) = E(X)

**16.** Suppose X has a Gamma (α, λ) distribution.

i) Derive the Moment Generating Function of X, from first principles and hence obtain its Cumulant Generating Function.

ii) Obtain an expression for the coefficient of skewness.

**17.** A bivariate random variable $X = (X1,X2)$ has the following moment generating function.

$$M_X(t_1, t_2) = \frac{1}{3}(1 + e^{(t_1+2t_2)} + e^{(2t_1+t_2)}).$$

Determine the covariance between $X1$ and $X2$.

**18.** Let the random variables $X$ and $Y$ have joint probability density function (pdf) given by

$$f(x,y) = \begin{cases} 2 \ ; 0 < x < y < 1 \\ 0, \quad otherwise. \end{cases}$$

i) Find the marginal pdf of Y.

ii) Find the conditional pdf of $X$ given $Y = y$.

iii) Calculate the conditional mean $E(X/\ Y = 1/2)$.

iv) Calculate the conditional variance $V(X/\ Y = 1/2)$.

**19.** A random variable X has a Pareto distribution with parameters α = 3 and λ = 4 and Y is a random variable such that:
$E\ (Y \mid X = x) = 2x + 5$ and $Var\ (Y \mid X = x) = x^2 + 3$

Calculate the unconditional standard deviation of Y.

**20.** The random variable X has probability density

$$f(x) = \begin{cases} 8x & if\ 0 < x < \frac{1}{2} \\ 0 & otherwise \end{cases}$$

and the random variable $Y$ is such that the conditional density of $Y/X = x$

is $f_{Y/_X}(y/X = x) = \begin{cases} \frac{1}{x} & if\ 0 < y < x \\ 0 & otherwise \end{cases}$

Find

i) The joint distribution of (X,Y)
ii) The marginal distribution of Y
iii) The mean and variance of Y, using (ii) above

**21.** Show that the probability generating function of binomial distribution with mean 12 and n = 20 is $G_x(t) = (0.4 + 0.6t)^{20}$

Deduce the moment generating function of the above distribution.

**22.** Poisson with mean 5. The time (in minutes) the call center officer takes, Y, to process x complaints is modelled as having a distribution with a conditional mean and variance given by:

E (Y|X=x) = 2x + 3 and V (Y|X=x) = x + 1

Calculate the unconditional variance of the time the call center officer takes to process complaints in a day.